Fluid Mechanics

Unit-I:

PROPERTIES OF FLUIDS

Fundamental Concepts:

Mechanics : Deals with action of forces on bodies at rest or in motion.

State of rest and Motion: They are relative and depend on the frame of reference. If the position with reference to frame of reference is fixed with time, then the body is said to be in a state of rest. Otherwise, it is said to be in a state of motion.

Scalar and heater quantities: Quantities which require only magnitude to represent them are called scalar quantities. Quantities whijch acquire magnitudes and direction to represent them are called vector quantities.

Eg: Mass, time internal, Distance traveled _ Scalars

Weight, Displacement, Velocity _ Vectors

Velocity and Speed: Rate of displacement is called velocity and Rate and distance travelled is called Speed.

Unit: m/s

Acceleration: Rate of change of velocity is called acceleration. Negative acceleration is called retardation.

Momentum: The capacity of a body to impart motion to other bodies is called momentum.

The momentum of a moving body is measured by the product of mass and velocity the moving body

Momentum = Mass x Velocity

Unit: Kgm/s

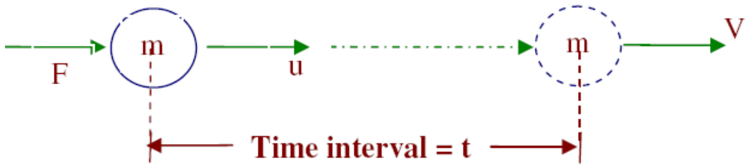
Newton's first law of motion: Every body continues to be in its state of rest or uniform motion unless compelled by an external agency.

Inertia: It is the inherent property the body to retain its state of rest or uniform motion.

Force: It is an external agency which overcomes or tends to overcome the inertia of a body.

Newton's second law of motion: The rate of change of momentum of a body is directly proportional to the magnitudes of the applied force and takes place in the direction of the applied force.

Measurement of force:



Change in momentum in time 't' = mv – mu

Rate of change of momentum = $\frac{mv - mu}{t}$

$$F \propto \frac{mv - mu}{t}$$

$$F \propto m\left(\frac{v - u}{t}\right)$$

$$F \propto ma$$

$$F = K\ ma$$

If F = 1 When m = 1 and u = 1

Then K = 1

F = ma.

Unit: newton (N)

Mass: Measure of amount of matter contained by the body it is a scalar quantity.

Unit: Kg.

Weight: Gravitational force on the body. It is a vector quantity.

F = ma

W = mg

Unit: newton (N) $g = 9.81\ m/s^2$

Volume: Measure of space occupied by the body.

Unit: m3

m3 = 1000 litres

Work: Work done = Force x Displacement _ Linear motion.

Work done = Torque x Angular displacement _ Rotatory motion.

Unit: Nm or J

Energy: Capacity of doing work is called energy.

Unit: Nm or J

Potential energy = mgh

Kinetic energy = ½ mv^2

Power: Rate of doing work is called Power.

$$\text{Power:} = \frac{\text{Force x displacement}}{\text{time}}$$

$$= \text{Force x Velocity} \rightarrow \text{Linear Motion.}$$

$$P = \frac{2\Pi N T}{60} \rightarrow \text{Rotatory Motion.}$$

Matter: Anything which possess mass and requires space to occupy is called matter.

States of matter:

Matter can exist in the following states

Solid state.

Fluid state.

Solid state: In case of solids intermolecular force is very large and hence molecules are not free to move. Solids exhibit definite shape and volume. Solids undergo certain amount of deformation and then attain state of equilibrium when subjected to tensile, compressive and shear forces.

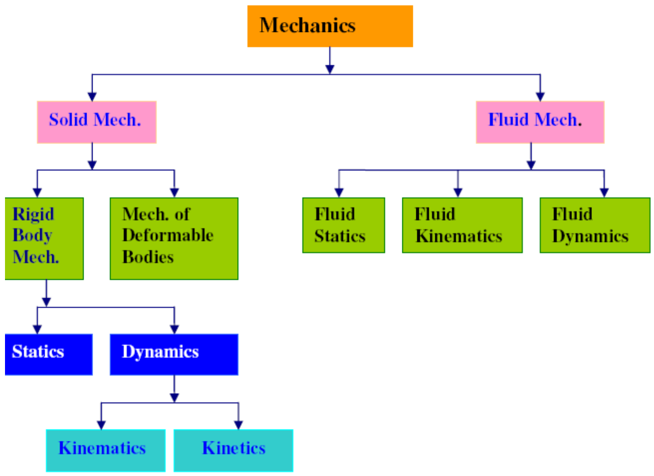
Fluid State: Liquids and gases together are called fluids. Incase of liquids Intermolecular force is comparatively small. Therefore liquids exhibit definite volume. But they assume the shape of the container

Liquids offer very little resistance against tensile force. Liquids offer maximum resistance against compressive forces. Therefore, liquids are also called incompressible fluids. Liquids undergo continuous or prolonged angular deformation or shear strain when subjected to tangential force or shear force. This property of the liquid is called flow of liquid. Any substance which exhibits the property of flow is called fluid. Therefore liquids are considered as fluids.

In case of gases intermolecular force is very small. Therefore the molecules are free to move along any direction. Therefore gases will occupy or assume the shape as well as the volume of the container.

Gases offer little resistance against compressive forces. Therefore gases are called compressible fluids. When subjected to shear force gases undergo continuous or prolonged angular deformation or shear strain. This property of gas is called flow of gases. Any substance which exhibits the property of flow is called fluid. Therefore gases are also considered as fluids.

Branches of Mechanics:



I. Fluid Statics deals with action of forces on fluids at rest or in equilibrium.

II. Fluid Kinematics deals with geometry of motion of fluids without considering the cause of motion.

III. Fluid dynamics deals with the motion of fluids considering the cause of motion.

<u>Properties of fluids:</u>

1. Mass density or Specific mass (ρ):

Mass density or specific mass is the mass per unit volume of the fluid.

$$\rho = \frac{Mass}{Volume}$$

$$\rho = \frac{M}{V} \text{ or } \frac{dM}{dV}$$

2. Weight density or Specific weight (ϒ):

Weight density or Specific weight of a fluid is the weight per unit volume.

Unit: kg/m3 or kgm3

With the increase in temperature volume of fluid increases and hence mass density decreases.

In case of fluids as the pressure increases volume decreases and hence mass density increases.

$$\therefore \quad \gamma = \frac{Weight}{Volume}$$

$$\gamma = \frac{W}{V} \text{ or } \frac{dW}{dV}$$

Unit: N/m3

$$\text{We have } \gamma = \frac{Weight}{Volume}$$

$$\gamma = \frac{mass \ x \ g}{Volume}$$

$$\gamma = \rho \text{ x g}$$

3. Specific gravity or Relative density (S):

It is the ratio of specific weight of the fluid to the specific weight of a standard fluid.

$$S = \frac{\gamma \text{ of fluid}}{\gamma \text{ of standard fluid}}$$

Unit: It is a dimensionless quantity and has no unit.

In case of liquids water at 4^0C is considered as standard liquid.

γ (specific weight) of water at 4^oC (standard liquid) is 9.81 $\frac{kN}{m^3}$ or 9.81 x 10^3 $\frac{kN}{m^3}$

Note: We have

1. $S = \frac{\gamma}{\gamma_{standard}}$

$\therefore \quad \gamma = S \times \gamma_{standard}$

2. $S = \frac{\gamma}{\gamma_{standard}}$

$S = \frac{\rho \times g}{\rho_{standard} \times g}$

$S = \frac{\rho}{\rho_{standard}}$

Specific gravity or relative density of a fluid can also be defined as the ratio of mass density of the fluid to mass density of the standard fluid. Mass density of standard water is 1000 kg/m3.

4. **Specific volume (∀):** It is the volume per unit mass of the fluid.

$$\therefore \quad \forall = \frac{Volume}{mass}$$

$$\forall = \frac{V}{M} \; or \; \frac{dV}{dM}$$

Unit: m3/kg

As the temperature increases volume increases and hence specific volume increases. As the pressure increases volume decreases and hence specific volume decreases.

Problems:

1. Calculate specific weight, mass density, specific volume and specific gravity of a liquid having a volume of 4m3 and weighing 29.43 kN. Assume missing data suitably.

$\gamma = ?$
$\rho = ?$
$\forall = ?$
$S = ?$
$V = 4\,m^3$
$W = 29.43\ kN$
$= 29.43 \times 10^3\,N$

$$\gamma = \frac{W}{V}$$

$$= \frac{29.43X10^3}{4}$$

$$\gamma = 7357.58\ N/m^3$$

To find ρ - Method 1:

$$W = mg$$

$$29.43 \times 10^3 = m \times 9.81$$

$$m = 3000\,kg$$

$$\therefore \rho = \frac{m}{v} = \frac{3000}{4}$$

$$\rho = 750\,kg/m^3$$

Method 2:

$$\gamma = \rho\ g$$

$$7357.5 = \rho\ 9.81$$

$$\rho = 750\ kg/m^3$$

$$\rho = \frac{M}{V}$$

i) $\forall = \dfrac{V}{M}$

$$= \frac{4}{3000}$$

$$\forall = 1.33 \times 10^{-3}\,m^3/kg$$

$$\forall = \frac{1}{\left(\frac{M}{V}\right)}$$

$$\forall = \frac{1}{\rho} = \frac{1}{750}$$

$$\forall = 1.33X10^{-3}\,m^3/kg$$

$$S = \frac{\gamma}{\gamma_{Standard}}$$

$$= \frac{7357.5}{9810}$$

$$S = 0.75$$

or

$$S = \frac{\rho}{\rho_{Standaard}}$$

$$S = \frac{750}{1000}$$

$$S = 0.75$$

2. Calculate specific weight, density, specific volume and specific gravity and if one litre of Petrol weighs 6.867N.

$$\gamma = \frac{W}{V}$$

$$= \frac{6.867}{10^{-3}}$$

$$\gamma = 6867 N/m^3$$

$V = 1 Litre = 10^{-3} m^3$

$W = 6.867 N$

$$S = \frac{\gamma}{\gamma_{Standard}}$$

$$= \frac{6867}{9810}$$

$$S = 0.7$$

$\rho = S\,g$

$6867 = \rho \times 9.81$

$\rho = 700 kg/m^3$

$$\forall = \frac{V}{M}$$

$$= \frac{10^{-3}}{0.7}$$

$$\forall = 1.4x10^{-3} m^3/kg$$

$M = W/g$

$M = 6.867 \div 9.81$

$M = 0.7$ kg

3. Specific gravity of a liquid is 0.7 Find i) Mass density ii) specific weight. Also find the mass and weight of 10 Litres of liquid.

$$S = \frac{\gamma}{\gamma_{Standard}}$$

$$0.7 = \frac{\gamma}{9810}$$

$$\gamma = 6867 N/m^3$$

$\gamma = \rho g$

$6867 = \rho \times 9.81$

$\rho = 700 kg/m^3$

$S = 0.7$
$V = ?$
$\rho = ?$
$M = ?$
$W = ?$
$V = 10$ litre
$= 10x10^{-3} m^3$

$$S = \frac{\rho}{\rho_{Standard}}$$

$$0.7 = \frac{\rho}{1000}$$

$$\rho = 700 kg/m^3$$

$$\rho = \frac{M}{V}$$

$$700 = \frac{M}{10x10^{-3}}$$

$$M = 7kg$$

$$\gamma = \frac{W}{V}$$

$$6867 = \frac{W}{10^{-2}}$$

$$W = 68.67 N$$

or

$$W = m\,g$$

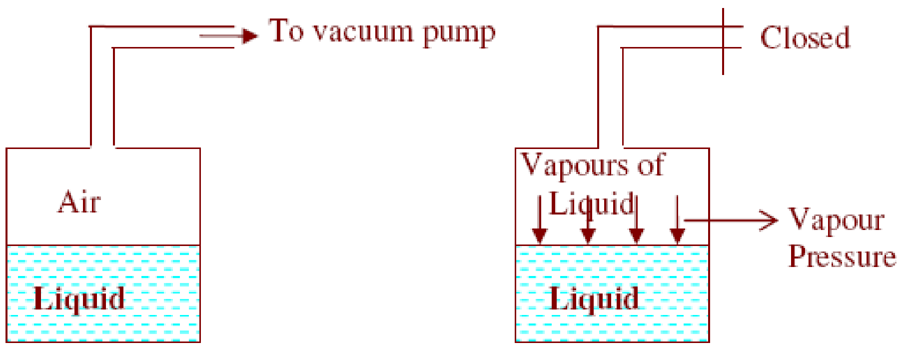
$$= 7 \times 9.81$$

$$W = 68.67 N$$

5. Vapour Pressure: The process by which the molecules of the liquid go out of its surface in the form of vapour is called Vaporisation. There are two ways of causing Vaporisation.

a. By increasing the temperature of the liquid to its boiling point.

b. By reducing the pressure above the surface of the liquid to a value less than Vapour pressure of the liquid.



As the pressure above the surface of the liquid is reduced, at some point, there will be vapourisation of the liquid. If the reduction in pressure is continued vapourisation will also continue. If the reduction in pressure is stopped, vapourisation continues until vapours of the liquid exert certain pressure which will just stop the vapourisation. This minimum partial pressure exerted by the

vapours of the liquid just to stop vapourisation is called Vapour Pressure of the liquid.
If the pressure over the surface goes below the vapour pressure, then, there will be vapourisation. But if the pressure above the surface is more than the vapour pressure then there will not be vapourisation unless there is heating.

Importance of Vapour Pressure:

In case of Hydraulic turbines sometimes pressure goes below the vapour pressure of

the liquid. This leads to vaporisation and formation of bubbles of liquid. When bubbles are carried to high Pressure zone they get busted leaving partial vacuum. Surrounding liquid enters this space with very high velocity exerting large force on the part of the machinery. This shenornenon is called cavitation. Turbines are designed such that there is no cavitation.

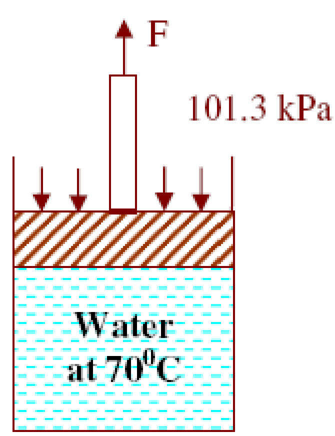
In Carburettors and sprayers vapours of liquid are created by reducing the pressure below vapour pressure of the liquid.

Unit of Vapour Pressure: N/m^2 (Pascal - Pa)

Vapour Pressure of a fluid increases with increase in temperature.

Problem

1. A vertical cylinder 300mm in diameter is fitted at the top with a tight but frictionless piston and filled with water at 700 C. The outer portion of the piston is exposed to atmospheric pressure of 101.3 kPa. Calculate the minimum force applied on the piston that will cause water to boil at 700 C. Take Vapour pressure of water at 70^0C as 32k Pa.



$D = 300$ mm

$= 0.3$ m

F Should be applied such that the Pressure is reduced from 101.3kPa to 32kPa.

There fore reduction in pressure required

$= 101.3 - 32$

$= 69.3$ kPa

$= 69.3 \times 10^3$ N/m^2

$\therefore$ F / Area $= 69.3 \times 10^3$

$$F / \frac{\Pi}{4} \times (0.3)^2 = 69.3 \times 10^3$$

$F = 4.9 \times 10^3$N

$F = 4.9$ kN

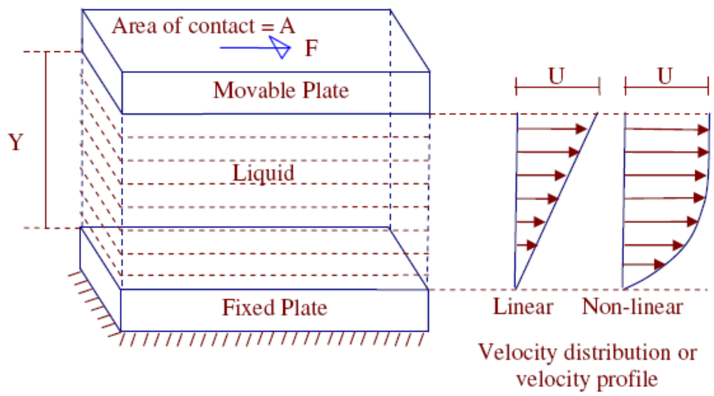
6. Viscosity:

Viscosity is the property by virtue of which fluid offers resistance against the flow or shear deformation. In other words, it is the reluctance of the fluid to flow. Viscous force is that force of resistance offered by a layer of fluid for the motion of another layer over it.

In case of liquids, viscosity is due to cohesive force between the molecules of adjacent layers of liquid. In case of gases, molecular activity between adjacent layers is the cause of viscosity.

1. Newton's law of viscosity:

Let us consider a liquid between the fixed plate and the movable plate at a distance 'Y' apart , 'A' is the contact area (Wetted area) of the movable plate , 'F' is the force required to move the plate with a velocity 'U' According to Newton



- $F \alpha A$
- $F \alpha \frac{1}{Y}$
- $F \alpha U$

$$\therefore F \alpha \frac{AU}{Y}$$

$$F = \mu . \frac{AU}{Y}$$

'μ' is the constant of proportionality called Dynamic Viscosity or Absolute Viscosity or Coefficient of Viscosity or Viscosity of the fluid.

$$\frac{F}{A} = \mu . \frac{U}{Y}$$

$$\therefore \tau = \mu \frac{U}{Y}$$

- τ is the force required; per unit area called 'Shear Stress'.

- The above equation is called Newton's law of viscosity.

Velocity gradient or rate of shear strain:

It is the difference in velocity per unit distance between any two layers.

If the velocity profile is linear then velocity gradient is given by U/Y. If the velocity profile is non – linear then it is given by du/dy.

- Unit of force (F): N.
- Unit of distance between the tow plates (Y): m
- Unit of velocity (U): m/s
- Unit of velocity gradient : $\frac{U}{Y} = \frac{m/s}{m} = /s = s^{-1}$
- Unit of dynamic viscosity (τ): $\tau = \mu \cdot \frac{u}{y}$

$$\mu = \frac{\tau y}{U}$$

$$\Rightarrow \frac{N/m^2 \cdot m}{m/s}$$

$$\mu \Rightarrow \frac{Ns}{m^2} \text{ or } \mu \Rightarrow P_a s$$

NOTE:

In CGS system unit of dynamic viscosity is $\frac{\text{dyne . Sec}}{\text{cm}^2}$ and is called poise (P).

If the value of μ is given in poise, multiply it by 0.1 to get it in $\frac{NS}{m^2}$.

1 Centipoise = 10^{-2} Poise.

2. Effect of Pressure on Viscosity of fluids:

Pressure has very little or no effect on the viscosity of fluids.

3. Effect of Temperature on Viscosity of fluids:

Effect of temperature on viscosity of liquids: Viscosity of liquids is due to cohesive force between the molecules of adjacent layers. As the temperature increases cohesive force decreases and hence viscosity decreases.

Effect of temperature on viscosity of gases: Viscosity of gases is due to molecular activity between adjacent layers. As the temperature increases molecular activity increases and hence viscosity increases.

4. **Kinematics Viscosity:** It is the ratio of dynamic viscosity of the fluid to its mass

density.

$$\therefore \text{Kinematic Viscosity} = \frac{\mu}{\rho}$$

Unit of KV:

$$KV \Rightarrow \frac{\mu}{\rho}$$

$$\Rightarrow \frac{NS/m^2}{kg/m^3}$$

$$= \frac{NS}{m^2} x \frac{m^3}{kg}$$

$$= \left(\frac{kg\,m}{s^2}\right) x \frac{s}{m^2} x \frac{m^3}{kg} = m^2/s$$

$$\therefore \text{Kinematic Viscosity} = m^2/s$$

$F = ma$
$N = Kg.m/s^2$

NOTE: Unit of kinematics viscosity in CGS system is cm^2/s and is called stoke (S)

If the value of KV is given in stoke, multiply it by 10^{-4} to convert it into m^2/s.

Problems:

1. Viscosity of water is 0.01 poise. Find its kinematics viscosity if specific gravity is 0.998.

Kinematics viscosity = ?

$S = 0.998$

$$S = \frac{\rho}{\rho_{standrad}}$$

$$0.998 = \frac{\rho}{1000}$$

$$\rho = 998 \text{ kg/m}^3$$

$$\mu = 0.01P$$

$$= 0.01 \times 0.1$$

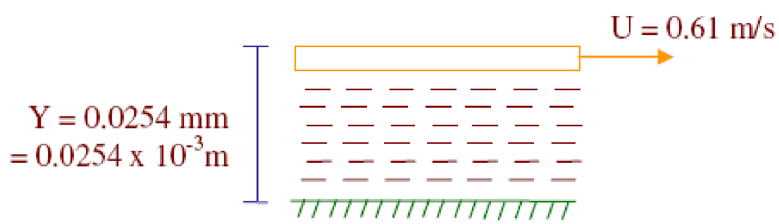
$$\mu = 0.001 \frac{NS}{m^2}$$

$$\therefore KV = \frac{\mu}{\rho}$$

$$= \frac{0.001}{998}$$

$$KV = 1 \times 10^{-6} \text{ m}^2/\text{s}$$

2. A Plate at a distance 0.0254mm from a fixed plate moves at 0.61m/s and requires a force of 1.962N/m2 area of plate. Determine dynamic viscosity of liquid between the plates.



$$\tau = 1.962 \text{ N/m}^2$$

$$\mu = ?$$

Assuming linear velocity distribution

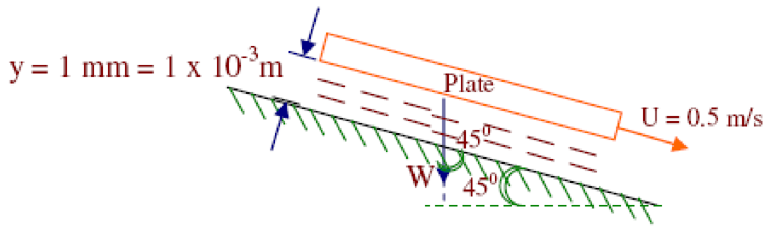
$$\tau = \mu \frac{U}{Y}$$

$$1.962 = \mu \times \frac{0.61}{0.0254 \times 10^{-3}}$$

$$\mu = 8.17 \times 10^{-5} \frac{NS}{m^2}$$

3. A plate having an area of 1m2 is dragged down an inclined plane at 450 to horizontal with a velocity of 0.5m/s due to its own weight. Three is a cushion of liquid 1mm thick between the inclined plane and the plate. If viscosity of oil is 0.1 Pas find the weight of the plate.

Sol:



$A = 1m^2$

$U = 0.5m/s$

$Y = 1x10^{-3}m$

$\mu = 0.1NS/m^2$

$W = ?$

$F = W \text{ x } \cos 45^0$

$\quad = W \text{ x } 0.707$

$F = 0.707W$

$$\tau = \frac{F}{A}$$

$$\tau = \frac{0.707W}{1}$$

$$\tau = 0.707 \text{ W N/m}^2$$

Assuming linear velocity distribution,

$$\tau = \mu.\frac{U}{Y}$$

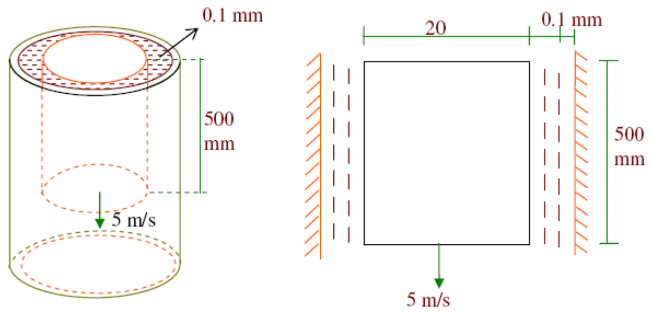
$$0.707W = 0.1 \text{ x } \frac{0.5}{1 \text{ x } 10^{-3}}$$

$$W = 70.72 \text{ N}$$

4. A shaft of φ 20mm and mass 15kg slides vertically in a sleeve with a velocity of 5 m/s. The gap

between the shaft and the sleeve is 0.1mm and is filled with oil. Calculate the viscosity of oil if the length of the shaft is 500mm.

Sol:



$D = 20mm = 20x10^{-3}m$

$M = 15$ kg

$W = 15x\ 9.81$

$\underline{W = 147.15N}$

$y = 0.1mm$

$\underline{y = 0.1 \text{ x } 10^{-3}mm}$

$U = 5m/s$

$F = W$

$\underline{F = 147.15N}$

$\mu = ?$

$A = \Pi\ D\ L$

$A = \Pi \text{ x } 20 \text{ x } 10^{-3} \text{ x } 0.5$

$\underline{A = 0.031\ m^2}$

$$\tau = \mu.\frac{U}{Y}$$

$$4746.7 = \mu\ x \frac{5}{0.1x10^{-3}}$$

$$\mu = 0.095\ \frac{NS}{m^2}$$

$$\tau = \frac{F}{A}$$

$$= \frac{147.15}{0.031}$$

$$\tau = 4746.7N / m^2$$

5. If the equation of velocity profile over 2 plate is V= 2y2/3 . in which 'V' is the velocity in m/s and 'y' is the distance in 'm' . Determine shear stress at (i) y = 0 (ii) y = 75mm. Take μ = 8.35P.

i. at y = 0

ii. at y = 75mm (75 x 10^{-3} m)

$$\tau = 8.35 \text{ P}$$

$$= 8.35 \text{ x } 0.1 \ \frac{NS}{m^2}$$

$$= 0.835 \frac{NS}{m^2}$$

$$V = 2y^{2/3}$$

$$\frac{dv}{dy} = 2x\frac{2}{3}y^{2/3-1}$$

$$= \frac{4}{3}y^{-1/3} = \frac{4}{3}\frac{1}{\sqrt[3]{y}}$$

$$\text{at, } y = 0, \frac{dv}{dy} = 3\frac{4}{\sqrt[3]{0}} = \infty$$

$$\text{at, } y = 75\text{x}10^{-3}\text{m}, \frac{dv}{dy} = 3\frac{4}{\sqrt[3]{75\text{x}10^{-3}}}$$

$$\frac{dv}{dy} = 3.16/\text{s}$$

$$\tau = \mu.\frac{dv}{dy}$$

$$at, y = 0, \ \tau = 0.835x\infty$$

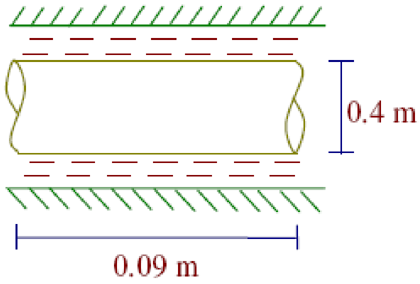
$$\tau = \infty$$

$$at, y = 75x10^{-3}m, \tau = 0.835 \ x \ 3.16$$

$$\tau = 2.64 \ N/m^2$$

6. Dynamic viscosity of oil used for lubrication between a shaft and a sleeve is 6 P. The shaft is of diameter

0.4 m and rotates at 190 rpm. Calculate the power lost in the bearing for a sleeve length of 0.09 m. Thickness of oil is 1.5 mm.



$\mu = 6P$

$= 0.6 \frac{Ns}{m^2}$

N = 190 rpm

Power lost = ?

$A = \Pi D L$

$= \Pi \times 0.4 \times 0.09 \quad A = 0.11 m^2$

$Y = 1.5 \times 10^{-3}$ m

$$U = \frac{\Pi DN}{60}$$

$$= \frac{\Pi \text{ x } 0.4 \text{ x } 190}{60}$$

$$U = 3.979 \text{ m/s}$$

$$\tau = \mu.\frac{U}{Y}$$

$$= 0.6 \text{ x } \frac{3.979}{1.5 \text{ x } 10^{-3}}$$

$$\tau = 1.592 \text{ x } 10^{3} \text{ N/m}^{2}$$

$$\frac{F}{A} = 1.59 \text{ x } 10^{3}$$

$$F = 1.591 \text{x } 10^{3} \text{x } 0.11$$

$$F = 175.01 \text{ N}$$

$$T = F \text{x } R$$

$$= 175.01 \text{x} 0.2$$

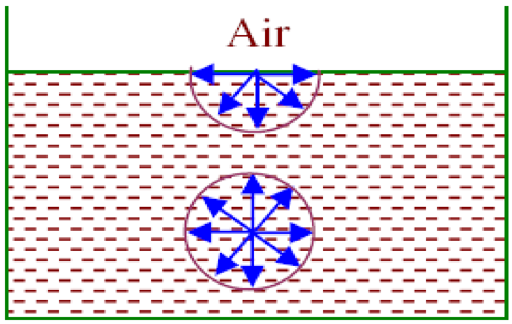
$$T = 35 \text{Nm}$$

$$P = \frac{2\Pi NT}{60{,}000}$$

$$P = 0.6964 \text{KW}$$

$$P = 696.4 \text{W}$$

(7) Surface Tension (σ):



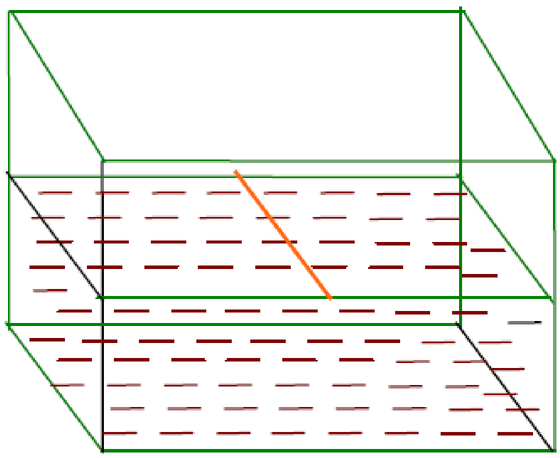
➢ Surface tension is due to cohesion between the molecules of liquid and weak adhesion between the molecules on the exposed surface of the liquid and molecules of air.

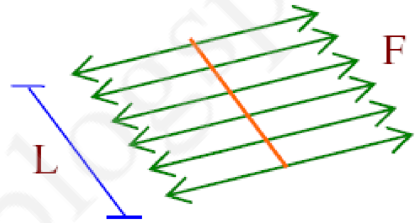
➢ A molecule inside the surface gets attracted by equal forces from the surrounding molecules whereas a molecule on the surface gets attracted by the molecule below it. Since there are no molecules above it, it experiences an unbalanced vertically downward force. Due to this entire surface of the liquid expose of to air will have a tendency to move in ward and hence the surface will be under tension. The property of the liquid surface to offer resistance against tension is called surface tension.

➢ **Consequences of Surface tension:**

- Liquid surface supports small loads.
- Formation of spherical droplets of liquid.
- Formation of spherical bubbles of liquid.
- Formation of cylindrical jet of liquids.

➢ Measurement of surface tension:





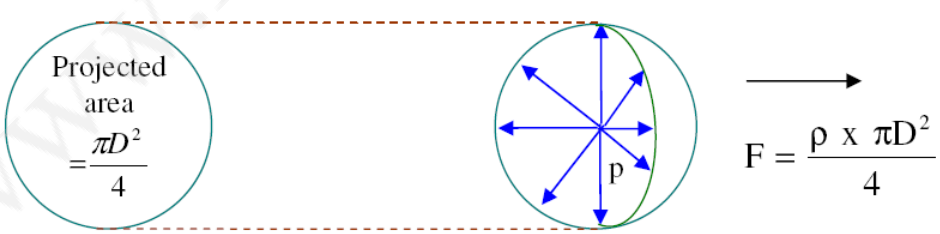
Surface tension is measured as the force exerted by the film on a line of unit length on the surfaceof the liquid. It can also be defined as the force required maintaining unit length of film in

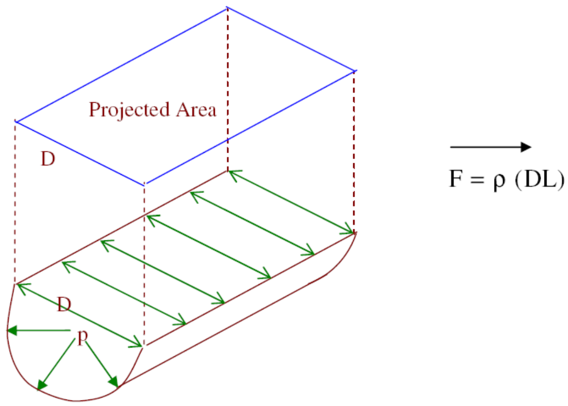
$\therefore \sigma = \frac{F}{L}$ $\qquad\qquad$ $\therefore F = \sigma L$

Unit: N/m

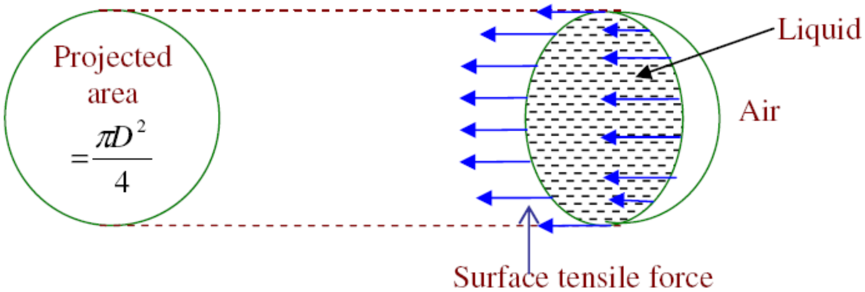
Force due to surface tension = σ x length of film

NOTE: Force experienced by a curved surface due to radial pressure is given by the product of intensity of pressure and projected area of the curved surface.





7.1 To derive an expression for the pressure inside the droplet of a liquid.



Let us consider droplet of liquid of surface tension σ, D is the diameter of the droplet. Let 'p' be the pressure inside the droplet in excess of outside pressure ($p = p_{inside} - p_{outside}$).

For the equilibrium of the part of the droplet,

Force due to surface tension = Force due to pressure

$$\sigma \times \Pi D = p \times \text{projected area}$$

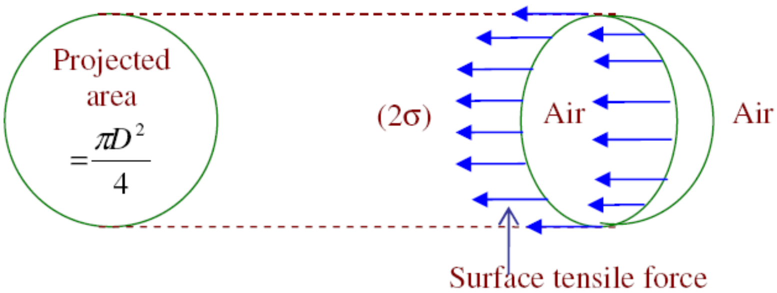
$$\sigma \times \Pi D = p \times \frac{\Pi D^2}{4}$$

$$p = \frac{4\sigma}{D}$$

As the diameter increases pressure decreases.

7.2 To derive an expression for the pressure inside the bubble of liquid:

`D' is the diameter of bubble of liquid of surface tension σ . Let 'p' be the pressure inside the bubble which is in excess of outside pressure. In case of bubble the liquid layer will be in contact with air both inside and outside.



For the equilibrium of the part of the bubble,

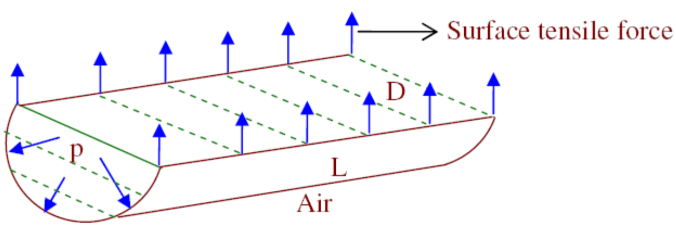
Force due to surface tension = Force due to pressure

$(2\sigma) \times \Pi D = p \times$ projected area

$$2[\sigma \times \Pi D] = p \times \frac{\Pi D^2}{4}$$

$$p = \frac{8\sigma}{D}$$

7.3 To derive an expression for the pressure inside the jet of liquid:



Let us consider a jet of diameter D of liquid of surface tension σ and p is the intensity of pressure inside the jet in excess of outside atmospheric pressure. For the equilibrium of the part of the jet shown in fig,

Force due to Radial pressure = Force due to surface tension p x Projected area = σ x Length

p x D x L = σ x 2L

$$P = \frac{2\sigma}{D}$$

- **Effect of temperature on surface tension of liquids:**

In case of liquids, surface tension decreases with increase in temperature. Pressure has no or very little effect on surface tension of liquids.

Problems:

1. What is the pressure inside the droplet of water 0.05 mm in diameter at 20^0C if the pressure outside the droplet is 103 kPa Take σ = 0.0736 N/m at 20^0C.

$$p = \frac{4\sigma}{D}$$

$$= \frac{4x0.0736}{0.05x10^{-3}}$$

$$p = 5.888x10^{+3}\,N/m^2$$

$$p = p_{inside} - p_{outside}$$

$$p_{inside} = (5.888 + 103)10^3$$

$$p_{inside} = 108.88x10^3\,Pa$$

$$p_{inside} = ?$$

$$D = 0.05x10^{-3}\,m$$

$$p_{outside} = 103kPa$$

$$= 103x10^3\,N/m^2$$

$$\sigma = 0.0736N/m$$

2. liquid bubble 2cm in radius has an internal pressure of 13Pa. Calculate the surface tension of liquid film.

$$p = \frac{8\sigma}{D}$$

$$\sigma = \frac{13x4x10^{-2}}{8}$$

$$\sigma = 0.065N/m$$

$$R = 2cm$$

$$D = 4cm$$

$$= 4x10^{-2}m$$

$$p = 13Pa(N/m^2)$$

Compressibility:

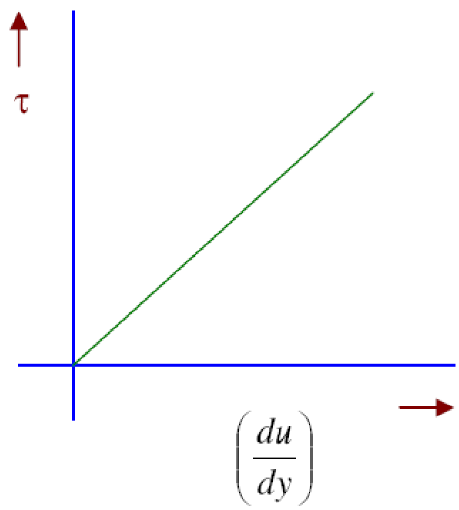
It is the property by virtue of which there will be change in volume of fluid due to change in pressure.

Rheological classification of fluids: (Rheology _ Study of stress – strain behavior).

1. **Newtonian fluids:** A fluid which obeys Newton's law of viscosity i.e., t = μ. *du/dy* is called Newtonian fluid. In such fluids shear stress varies directly as shear strain.

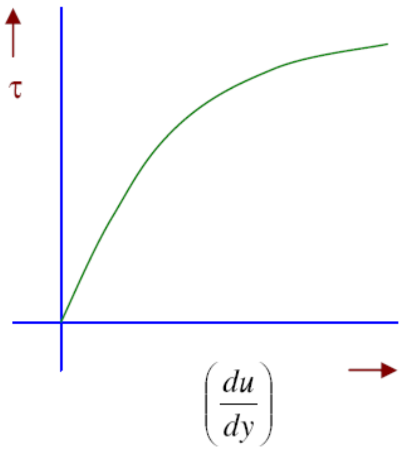
In this case the stress strain curve is a stress line passing through origin the slope of the line gives dynamic viscosity of the fluid.

Eg: Water, Kerosene.



3. **Non- Newtonian fluid:** A fluid which does not obey Newton's law of viscosity is called non-Newton fluid. For such fluids,

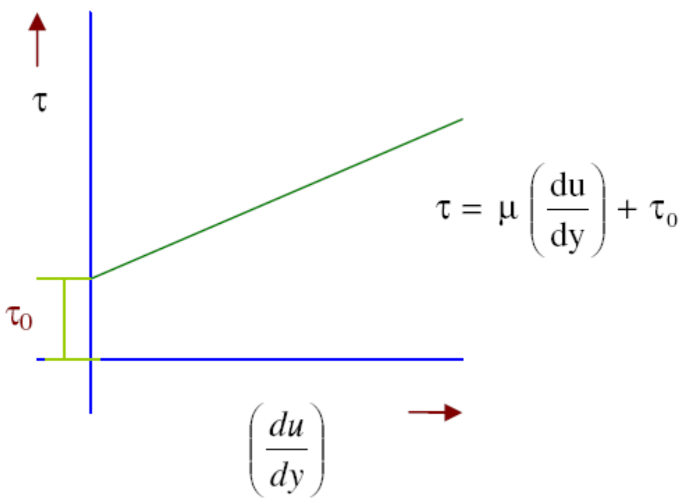
$$\tau = \mu . \left(\frac{du}{dy}\right)^n$$



3. **Ideal Plastic fluids:**

In this case the strain starts after certain initial stress (t0) and then the stress strain relationship will be linear. t0 is called initial yield stress. Sometimes they are also called Bingham's Plastics.

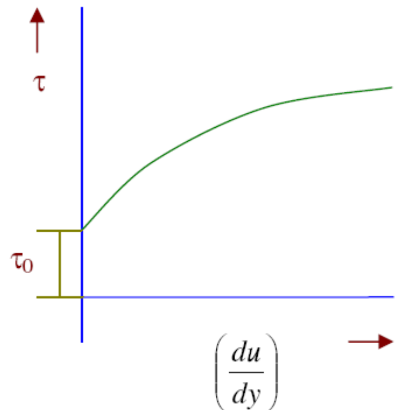
Eg: Industrial sludge.



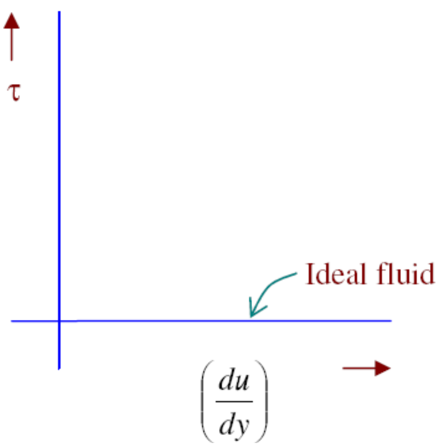
4. **Thixotropic fluids:**

These require certain amount of yield stress to initiate shear strain. After wards stress-strain relationship will be non – linear.

Eg; Printers ink.

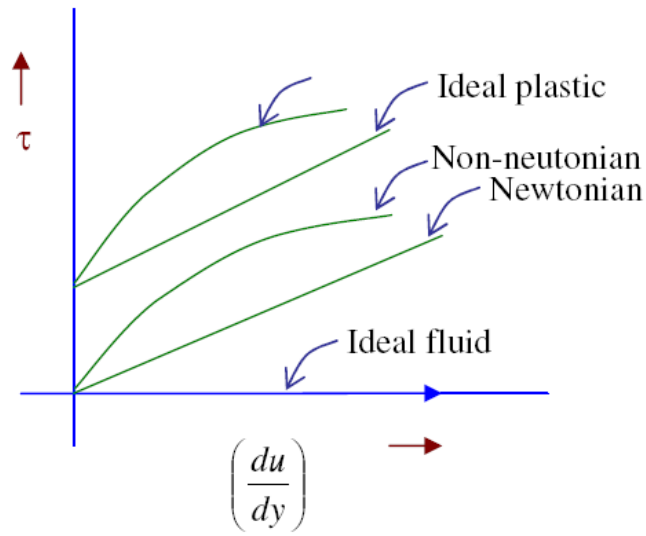


5. **Ideal fluid:** Any fluid for which viscosity is assumed to be zero is called Ideal fluid. For ideal fluid t = 0 for all values of du/dy

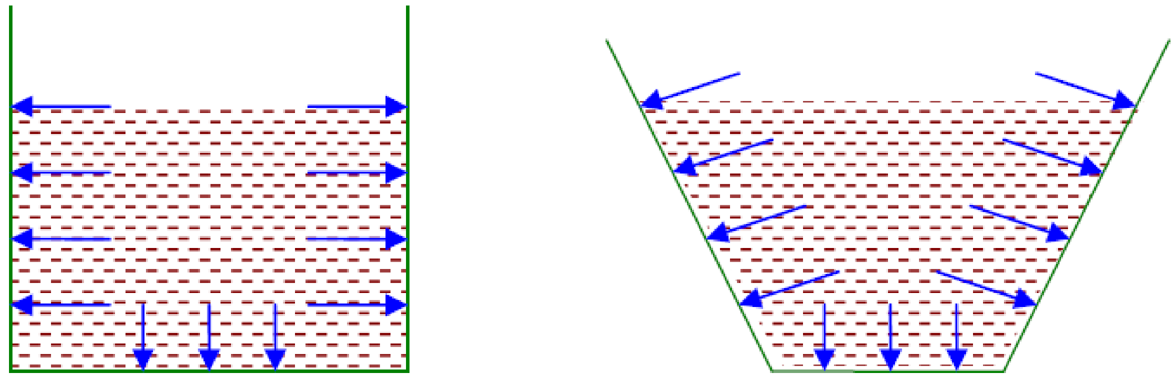


6. **Real fluid :**

Any fluid which posses certain viscosity is called real fluid. It can be Newtonian or non – Newtonian, thixotropic or ideal plastic.



PRESSURE AND ITS MEASUREMENTS:



Fluid is a state of matter which exhibits the property of flow. When a certain mass of fluids is held in static equilibrium by confining it within solid boundaries, it exerts force along direction perpendicular to the boundary in contact. This force is called fluid pressure.

• **Pressure distribution:**

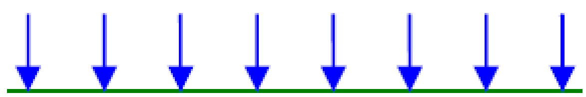
It is the variation of pressure over the boundary in contact with the fluid.

There are two types of pressure distribution.

a) Uniform Pressure distribution.

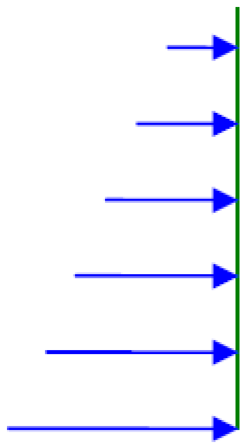
b) Non-Uniform Pressure distribution.

(a) Uniform Pressure distribution:



If the force exerted by the fluid is same at all the points of contact boundary then the pressure distribution is said to be uniform.

(b) Non –Uniform Pressure distribution:



If the force exerted by the fluid is not same at all the points then the pressure distribution is said to be non-uniform.

• **Intensity of pressure or unit pressure or Pressure:**

Intensity of pressure at a point is defined as the force exerted over unit area considered around that point. If the pressure distribution is uniform then intensity of pressure will be same at all the points.

• **Calculation of Intensity of Pressure:**

When the pressure distribution is uniform, intensity of pressure at any points is given by the ratio of total force to the total area of the boundary in contact.

Intensity of Pressure ‘p’ $=F/A$

When the pressure distribution is non- uniform, then intensity of pressure at a point is given by dF/dA.

Unit of Intensity of Pressure: N/m^2 or pascal (Pa).

Note: 1 MPa $=1N/mm^2$

• **Atmospheric pressure**

Air above the surface of liquids exerts pressure on the exposed surface of the liquid and normal to the surface.

This pressure exerted by the atmosphere is called atmospheric pressure.

Atmospheric pressure at a place depends on the elevation of the place and the temperature.

Atmospheric pressure is measured using an instrument called 'Barometer' and hence atmospheric pressure is also called Barometric pressure.

Unit: kPa .

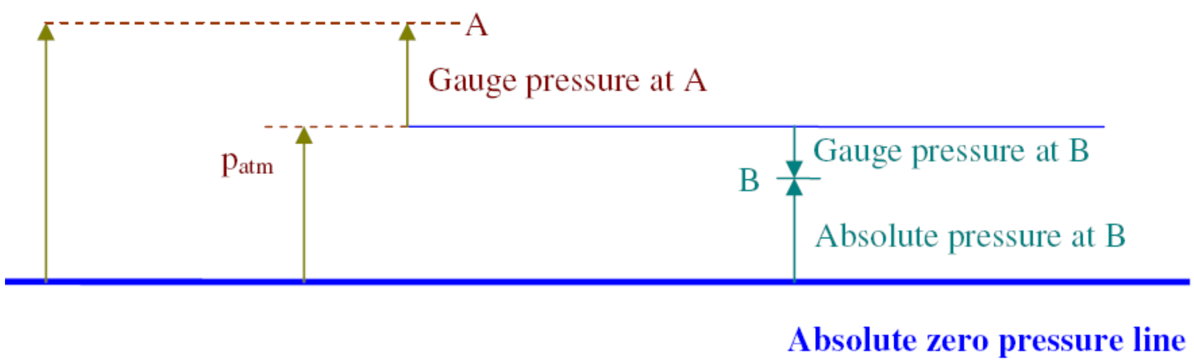
'bar' is also a unit of atmospheric pressure 1bar = 100 kPa.

• **Absolute pressure and Gauge Pressure:**

Absolute pressure at a point is the intensity of pressure at that point measured with reference to absolute vacuum or absolute zero pressure.

Absolute pressure at a point can never be negative since there can be no pressure less than absolute zero pressure.

Absolute pressure at 'A'



Absolute pressure at a point is the intensity of pressure at that point measured with reference to absolute vacuum or absolute zero pressure.

Absolute pressure at a point can never be negative since there can be no pressure less than absolute zero pressure.

If the intensity of pressure at a point is measured with reference to atmospheric pressure, then it is called gauge pressure at that point.

Gauge pressure at a point may be more than the atmospheric pressure or less than the atmospheric pressure. Accordingly gauge pressure at the point may be positive or negative.

Negative gauge pressure is also called vacuum pressure.

From the figure, It is evident that, Absolute pressure at a point = Atmospheric pressure ± Gauge pressure.

NOTE: If we measure absolute pressure at a Point below the free surface of the liquid, then,

p = g. Y + patm

If gauge pressure at a point is required, then atmospheric pressure is taken as zero, then,

p = g. Y

Pressure Head

It is the depth below the free surface of liquid at which the required pressure intensity is available.

P = gh

h = *P/* g

For a given pressure intensity 'h' will be different for different liquids since, 'g' will be different for different liquids. Whenever pressure head is given, liquid or the property of liquid like specify gravity, specific weight, mass density should be given.

Eg:

(i) 3m of water

(ii) 10m of oil of S = 0.8.

(iii) 3m of liquid of g = 15 kN/m3

(iv) 760mm of Mercury.

(v) 10m _ not correct.

NOTE:

1. To convert head of a liquid to head of another liquid.

$$S = \frac{\gamma}{\gamma_{Standard}}$$

$$S_1 = \frac{\gamma_1}{\gamma_{Standard}}$$

$$p = \gamma_1 h_1$$

$$\therefore \gamma_1 = S_1 \gamma_{Standard}$$

$$p = \gamma_2 h_2$$

$$\gamma_{21} = S_2 \gamma_{Standard}$$

$$\boxed{\gamma_1 h_1 = \gamma_2 h_2}$$

$$\therefore S_1 \ \gamma_{Standard} \ h_1 = S_2 \ \gamma_{Standard} \ h_2$$

$$\boxed{S_1 h_1 = S_2 h_2}$$

2. Swater x hwater = Sliquid x hliquid

1x hwater = Sliquid x hliquid

h water = Sliquid x hliquid

Pressure head in meters of water is given by the product of pressure head in meters of liquid and specific gravity of the liquid.

Eg: 10meters of oil of specific gravity 0.8 is equal to 10x0.8 = 8 meters of water.

Eg: Atmospheric pressure is 760mm of Mercury.

NOTE:

P	=	g	h
kPa		kN/m^3	m

Problem:

1. Calculate intensity of pressure due to a column of 0.3m of (a) water (b) Mercury (c) Oil of specific gravity-0.8.

a) h = 0.3m of water

$\gamma = 9.81 \, \frac{kN}{m^3}$

$p = ?$

$p = \gamma h$

$p = 2.943$ kPa

c) $h = 0.3$ of Hg

$\gamma = 13.6 \times 9.81$

$\gamma = 133.416 \text{ kN/m}^3$

$p = \gamma \; h$

$= 133.416 \times 0.3$

$p = 40.025$ kPa

2. Intensity of pressure required at a points is 40kPa. Find corresponding head in (a) water (b) Mercury (c) oil of specific gravity-0.9.

(a) p = 40 kPa

$h = \frac{p}{\gamma}$

$h = 4.077\,\text{m of water}$

$\gamma = 9.81 \frac{kN}{m^3}$

h = ?

(b) p = 40 kPa

$\gamma = (13.6x9.81\ N/m^3$

$\gamma = 133.416\ \frac{KN}{m^3}$

$h = \frac{p}{\gamma}$

$h = 0.299\,\text{m of Mercury}$

c) p = 40 kPa

$h = \frac{p}{\gamma}$

$h = 4.53\ \text{m of oil}\ S = 0.9$

$\gamma = 0.9\ x\ 9.81$

$\gamma = 8.829\ \frac{KN}{m^3}$

4. Standard atmospheric pressure is 101.3 kPa Find the pressure head in (i) Meters of water (ii) mm of mercury (iii) m of oil of specific gravity 0.8.

(i) $p = \gamma h$

$101.3 = 9.81\ x\ h$

h = 10.3 m of water

(ii) $p = \gamma h$

$101.3 = (13.6x9.81)\ x\ h$

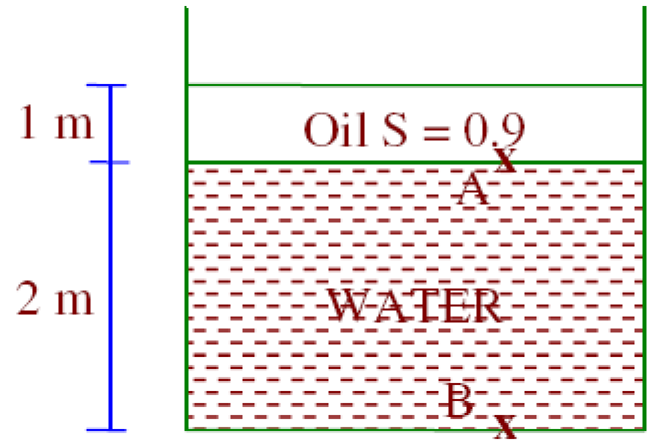
h = 0.76 m of mercury

(iii) $p = \gamma h$

$101.3 = (0.8\ x\ 9.81\ x\ h$

h = 12.9 m of oil of S = 0.8

5. An open container has water to a depth of 2m and above this an oil of S = 0.9 for a depth of 1m. Find the intensity of pressure at the interface of two liquids and at the bottom of the tank.



$$p_A = \gamma_{a1}\ h_{oil}$$

$$= (0.9\ x\ 9.81)\ x\ 1$$

$$p_A = 8.829\ kPa$$

$$p_B = \gamma_{oil}\ xh_{oil} + \gamma_{water} + h_{water}$$

$$p_A = 8.829\,kPa + 9.81\ x\ 2$$

$$p_B = 28.45\,kPa$$

6. Convert the following absolute pressure to gauge pressure (a) 120kPa (b) 3kPa (c) 15m of H2o (d) 800mm of Hg.

(a) $p_{abs} = p_{atm} + p_{gauge}$

$\therefore\ p_{gauge} = p_{abs} - p_{atm} = 120 - 101.3 = 18.7$ kPa

(b) $p_{gauge} = 3\text{-}101.3 = -98.3$ kPa

$p_{gauge} = 98.3$ kPa (vacuum)

(c) $h_{abs} = h_{atm} + h_{gauge}$

$15 = 10.3 + h_{gauge}$

$h_{gauge} = 4.7$m of water

(d) $h_{abs} = h_{atm} + h_{gauge}$

$800 = 760 + h_{gauge}$

$h_{gauge} = 40$ mm of mercury

Measurement of Pressure

Various devices used to measure fluid pressure can be classified into,

1. Manometers

2. Mechanical gauges.

Manometers are the pressure measuring devices which are based on the principal of balancing the column of the liquids whose pressure is to be measured by the same liquid or another liquid. Mechanical gauges consist of an elastic element which deflects under the action of applied pressure and this movement will operate a pointer on a graduated scale.

Classification of Manometers:

Manometers are broadly classified into

a) Simple Manometers

b) Differential Manometers.

a) Simple Manometers

Simple monometers are used to measure intensity of pressure at a point.

They are connected to the point at which the intensity of pressure is required. Such a point is called gauge point.

b) Differential Manometers

Differential manometers are used to measure the pressure difference between two points. They are connected to the two points between which the intensity of pressure is required.

Types of Simple Manometers

Common types of simple manometers are

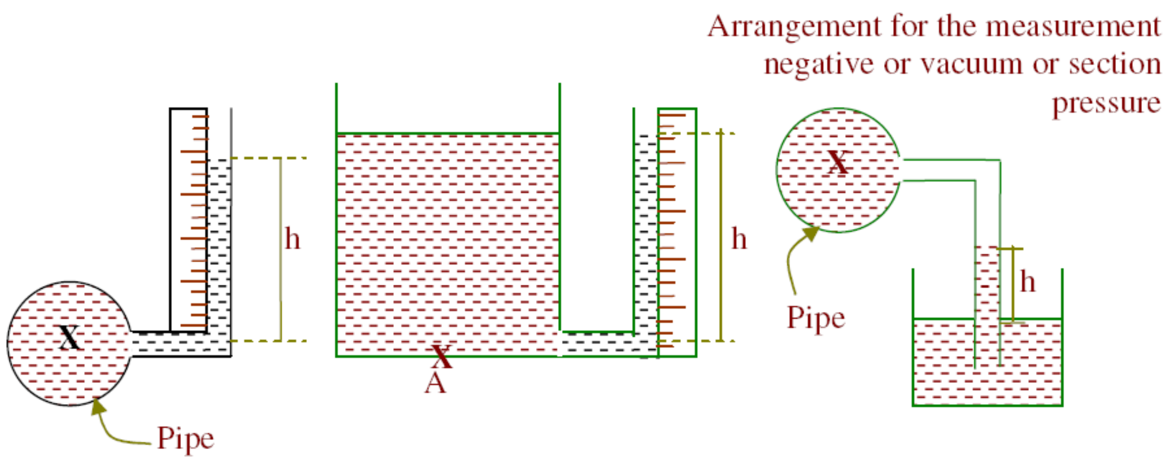
a) Piezometers

b) U-tube manometers

c) Single tube manometers

d) Inclined tube manometers

a) Piezometers:



Piezometer consists of a glass tube inserted in the wall of the vessel or pipe at the level of point at which the intensity of pressure is to be measured. The other end of the piezometer is exposed to air. The height of the liquid in the piezometer gives the pressure head from which the intensity of pressure can be calculated.

To minimize capillary rise effects the diameters of the tube is kept more than 12mm.

Merits

_ Simple in construction

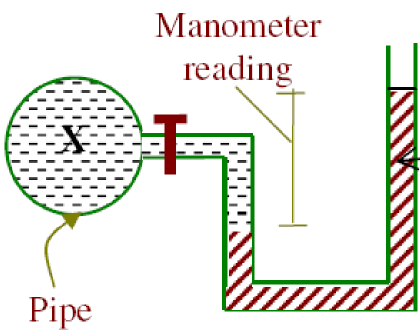
_ Economical

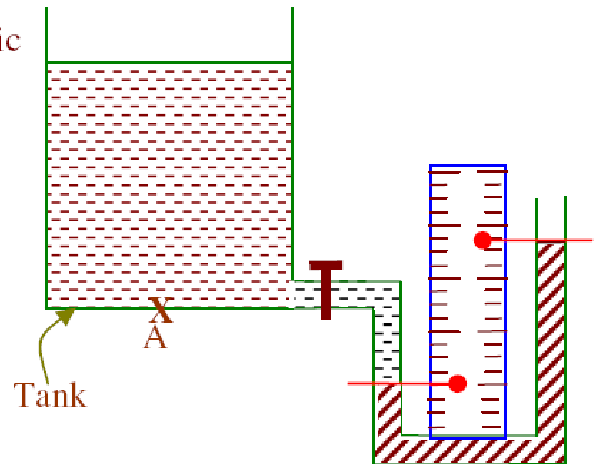
Demerits

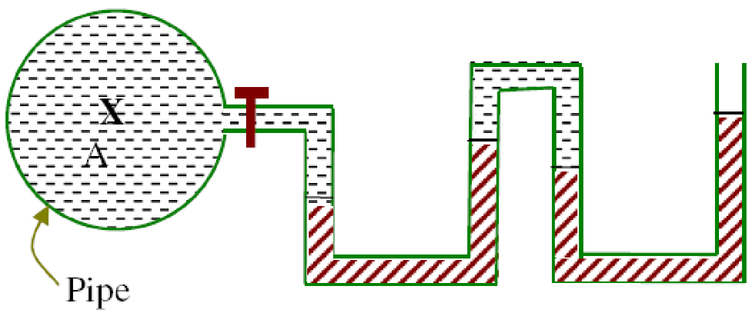
_ Not suitable for high pressure intensity.

_ Pressure of gases cannot be measured.

(b) U-tube Manometers:







A U-tube manometers consists of a glass tube bent in U-Shape, one end of which is connected to gauge point and the other end is exposed to atmosphere. U-tube consists of a liquid of specific of gravity other than that of fluid whose pressure intensity is to be measured and is called manometric liquid.

• **Manometric liquids**

¨ Manometric liquids should neither mix nor have any chemical reaction with the fluid whose pressure intensity is to be measured.

¨ It should not undergo any thermal variation.

¨ Manometric liquid should have very low vapour pressure.

¨ Manometric liquid should have pressure sensitivity depending upon the magnitude of pressure to be measured and accuracy requirement.

• **To write the gauge equation for manometers**

Gauge equations are written for the system to solve for unknown quantities.

Steps:

1. Convert all given pressure to meters of water and assume unknown pressure in meters of waters.

2. Starting from one end move towards the other observing the following points.

¨ Any horizontal movement inside the same liquid will not cause change in pressure.

¨ Vertically downward movement causes increase in pressure and upward motion causes decrease in pressure.

¨ Convert all vertical columns of liquids to meters of water by multiplying them by corresponding specify gravity.

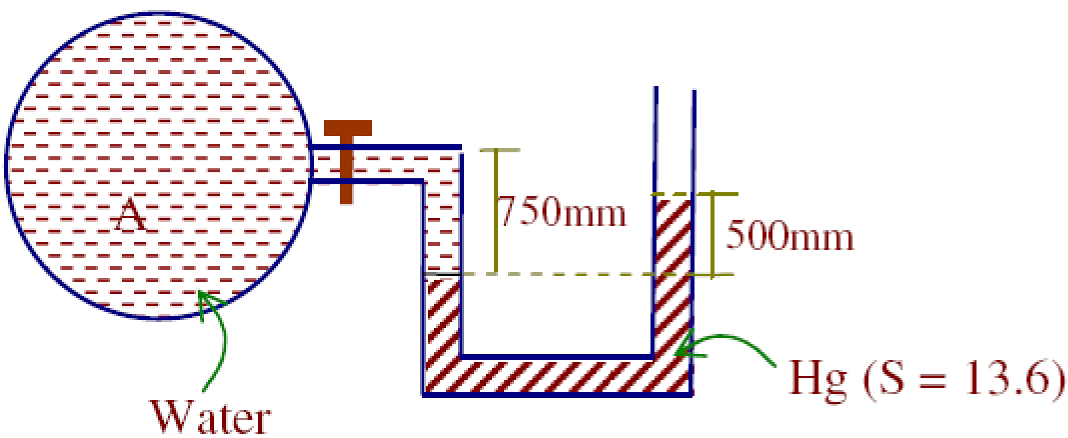
¨ Take atmospheric pressure as zero (gauge pressure computation).

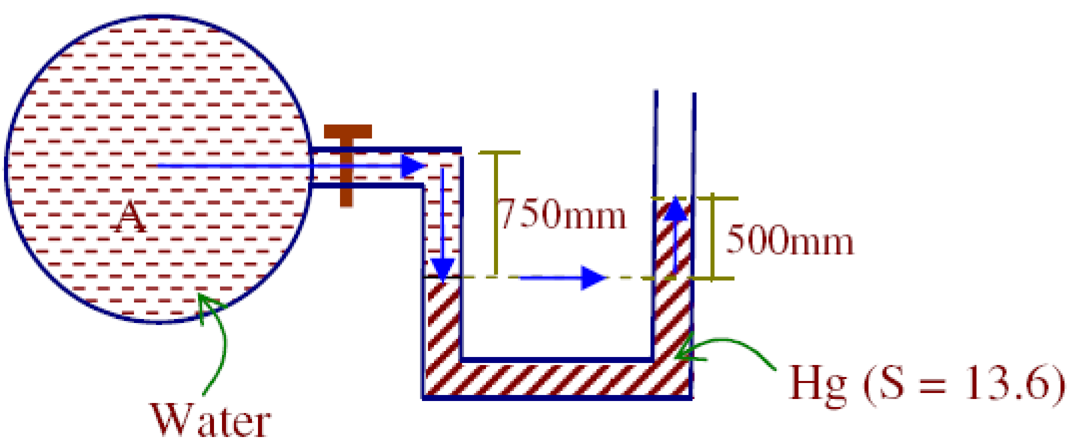
3. Solve for the unknown quantity and convert it into the required unit.

4. If required calculate absolute pressure.

Problem:

1. Determine the pressure at A for the U- tube manometer shown in fig. Also calculate the absolute pressure at A in kPa.





Let ‘h_A’ be the pressure head at ‘A’ in ‘meters of water’.

$$h_A + 0.75 - 0.5 x 13.6 = 0$$

$$h_A = 6.05 m \ of \ \ water$$

$$p = \gamma h$$

$$= 9.81 x 6.05$$

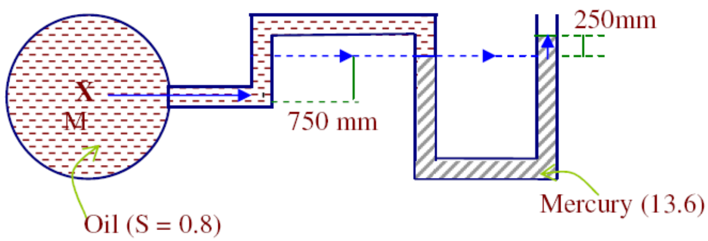
$$p = 59.35 \ kPa (gauge \ \ pressure)$$

$$p_{abs} = p_{atm} + p_{gauge}$$

$$= 101.3 + 59.35$$

$$p_{abc} = 160.65 \ kPa$$

2. For the arrangement shown in figure, determine gauge and absolute pressure at the point M.



Let ‘h_M’ be the pressure head at the point ‘M’ in m of water,

h_M - 0.75 x 0.8 – 0.25 x 13.6 = 0

h_M = 4 m of water

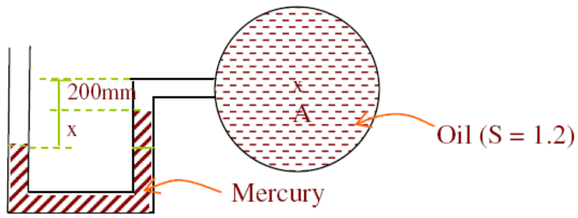
$p = \gamma h$

$p = 39.24 \text{kPa}$

$p_{abs} = 101.3 + 39.24$

$p_{abs} 140.54 \text{kPa}$

3. If the pressure at ‘At’ is 10 kPa (Vacuum) what is the value of ‘x’?



p_A = 10 kPa (Vacuum)

p_A = - 10 kPa

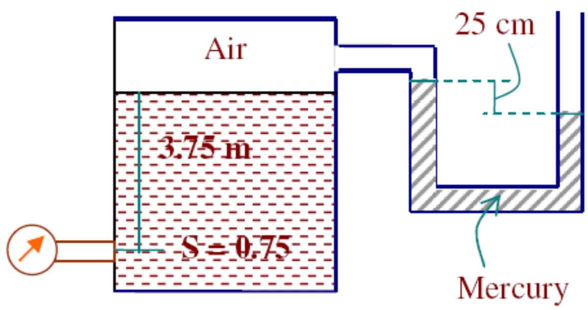
$$\frac{p_A}{\gamma} = \frac{-10}{9.81} = -1.019 \text{ m of water}$$

$$h_A = -1.019 \text{ m of water}$$

$$-1.019 + 0.2 \times 1.2 + x\,(13.6) = 0$$

$$x = 0.0572 \text{ m}$$

4. The tank in the accompanying figure consists of oil of S = 0.75. Determine the pressure gauge reading in *kN/m*^{2}.



Let the pressure gauge reading be 'h' m of water

h – 3.75 x 0.75 + 0.25 x 13.6 = 0

h = - 0.5875 m of water

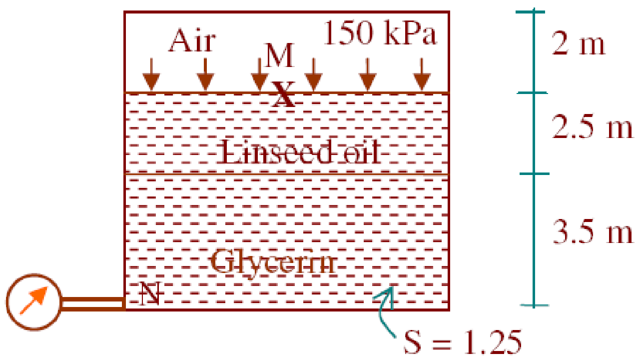
p = γ h

p = -5.763 kPa

p = 5.763 kPa (Vacuum)

5. A closed tank is 8m high. It is filled with Glycerine up to a depth of 3.5m and linseed oil to another 2.5m. The remaining space is filled with air under a pressure of 150 kPa. If a pressure gauge is fixed at the bottom of the tank what will be its reading.

Also calculate absolute pressure. Take relative density of Glycerine and Linseed oil as 1.25 and 0.93 respectively.



$$P_H = 150\,\text{kPa}$$

$$h_M = \frac{150}{9.81}$$

$$h_M = 15.29\ \text{m of water}$$

Let 'h_N' be the pressure gauge reading in m of water.

h_N -3.5 x 1.25 -2.5 x 0.93 =15.29

h_N = 21.99 m of water

p = 9.81 x 21.99

p = 215.72 kPa (gauge)

p_{abs} = 317.02 kPa

DIFFERENTIAL MANOMETERS

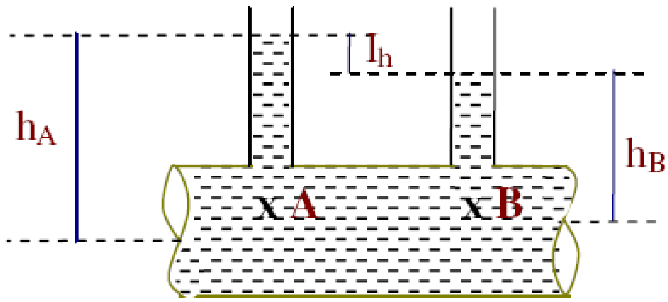
Differential manometers are used to measure pressure difference between any two points. Common varieties of differential manometers are:

(a) Two piezometers.

(b) Inverted U-tube manometer.

(c) U-tube differential manometers.

(d) Micromanometers.

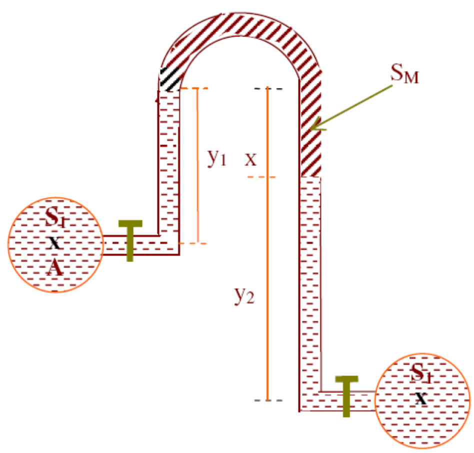


(a) Two Pizometers

The arrangement consists of two pizometers at the two points between which the pressure difference is required. The liquid will rise in both the piezometers. The difference in elevation of liquid levels can be recorded and the pressure difference can be calculated.

It has all the merits and demerits of piezometer.

(b) Inverted U-tube manometers



Inverted U-tube manometer is used to measure small difference in pressure between any two points. It consists of an inverted U-tube connecting the two points between which the pressure difference is required. In between there will be a lighter manometric liquid. Pressure difference between the two points can be calculated by writing the gauge equations for the system.

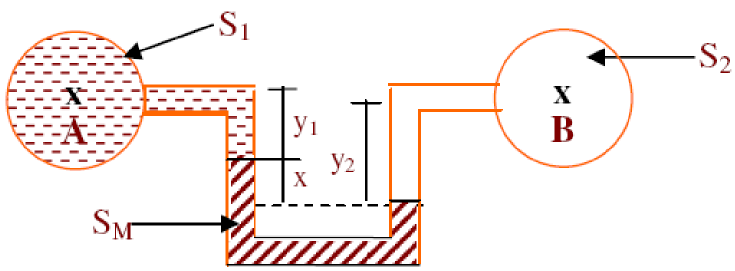
Let 'hA' and 'hB' be the pr head at 'A' and 'B' in meters of water

$h_A - (Y_1 S_1) + (x S_M) + (y_2 S_2) = h_B$.

$h_A - h_B = S_1 y_1 - S_M x - S_2 y_2$,

$p_A - p_B = g (h_A - h_B)$

(c) U-tube Differential manometers



A differential U-tube manometer is used to measure pressure difference between any two points. It consists of a U-tube containing heavier manometric liquid, the two limbs of which are connected to the gauge points between which the pressure difference is required. U-tube differential manometers can also be used for gases. By writing the gauge equation for the system pressure difference can be determined.

Let 'hA' and 'hB' be the pressure head of 'A' and 'B' in meters of water

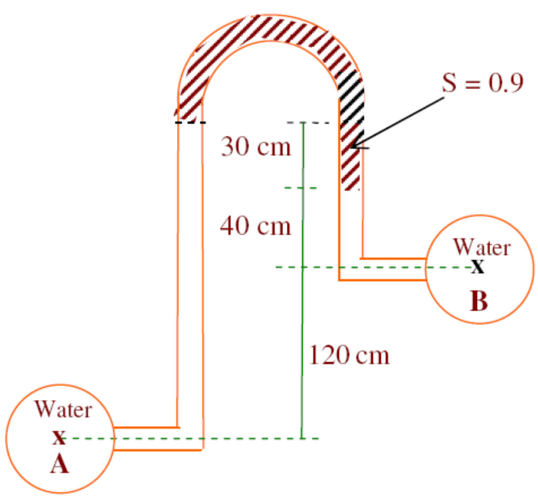
hA + S1 Y1 + x SM – Y2 S2 = hB

hA – hB = Y2 S2 – Y1 S1– x SM

Problems

(1) An inverted U-tube manometer is shown in figure. Determine the pressure difference between A and B in N/m^2

Let hA and hB be the pressure heads at A and B in meters of water.



$h_A - (190 \times 10^{-2}) + (0.3 \times 0.9) + (0.4)\ 0.9 = h_B$

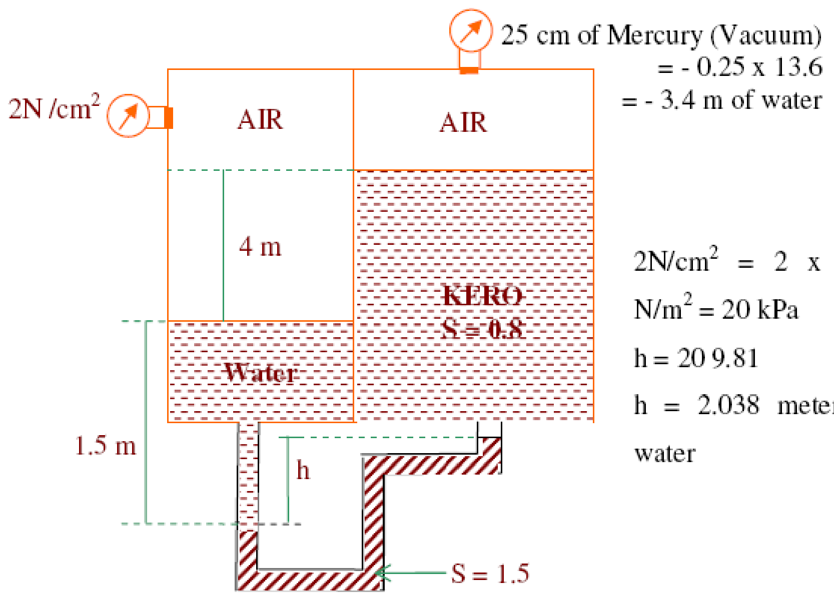
$h_A - h_B = 1.23$ meters of water

$p_A - p_B = \gamma\,(h_A - h_B) = 9.81 \times 1.23$

$p_A - p_B = 12.06$ kPa

$p_A - p_B = 12.06 \times 10^3\ N/m^2$

2. In the arrangements shown in figure. Determine the ho 'h'.



$2N/cm^2 = 2 \times 100^2\ N/m^2 = 20$ kPa

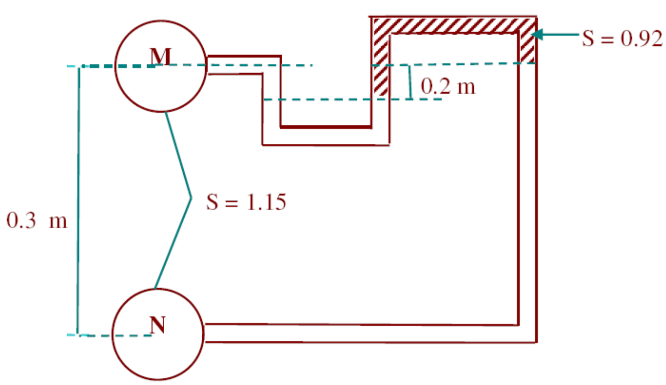
h = 20 9.81

h = 2.038 meters of water

$2.038 + 1.5 - (4 + 1.5 - h)\ 0.8 = -3.4$

h = 3.6 m

3. Compute the pressure different between 'M' and 'N' for the system shown in figure.



Let 'h_M' and ' h_N' be the pressure heads at M and N in m of water.

hm + y x 1.15 – 0.2 x 0.92 + (0.3 – y + 0.2) 1.15 = hn

hm + 1.15 y – 0.184 + 0.3 x 1.15 – 1.15 y + 0.2 x 1.15 = hn

hm + 0.391 = hn

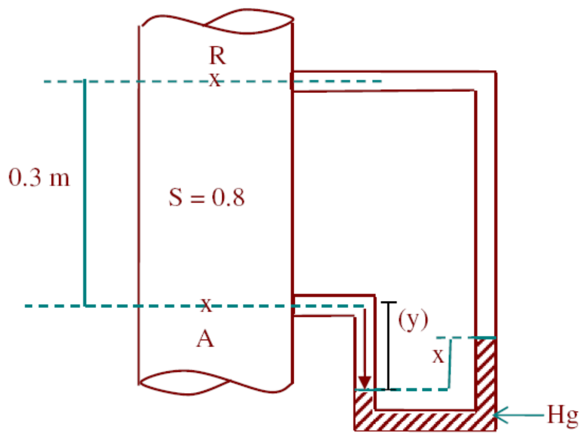
hn – hm = 0.391meters of water

$p_n - p_m = \gamma (h_N - h_m)$

= 9.81 x 0.391

$p_n - p_m$ = 3.835 kPa

4. Petrol of specify gravity 0.8 flows up through a vertical pipe. A and B are the two points in the pipe, B being 0.3 m higher than A. Connection are led from A and B to a U–tube containing Mercury. If the pressure difference between A and B is 18 kPa, find the reading of manometer.



$p_A - p_B$ = 18kPa

$$\frac{P_A - P_B}{\gamma}$$

$$h_A - h_B = \frac{18}{9.81}$$

$$h_A - h_B = 1.835 m \ of \ water$$

h_A + y x 0.8 – x 13.6 – (0.3 + y – x) 0.8 = h_B

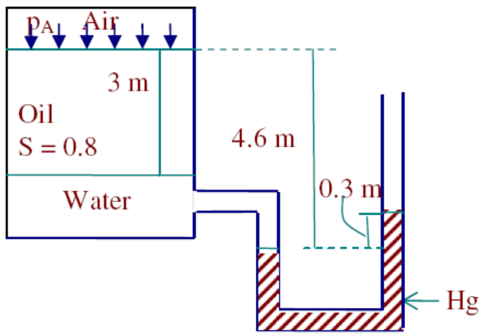
$h_A - h_B$ = – 0.8y + 13.66 x + 0.24 + 0.8 y – 0.8 x

$$h_A - h_B = 12.8x + 0.24$$

1.835 = 12.8x + 0.24

x = 0.1246 m

4. What is the pressure pA in the fig given below? Take specific gravity of oil as 0.8.



$h_A + (3 \times 0.8) + (4.6 - 0.3)(13.6) = 0$

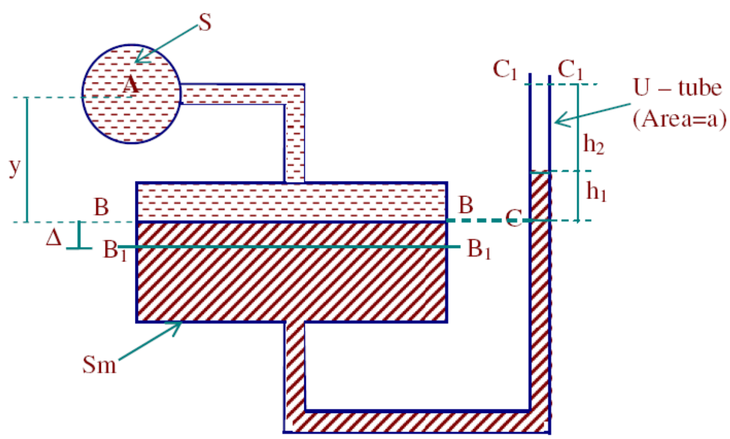
$h_A = 2.24$ m of oil

$p_A = 9.81 \times 2.24$

$p_A = 21.97$ kPa

SINGLE COLUMN MANOMETER:

Single column manometer is used to measure small pressure intensities.



A single column manometer consists of a shallow reservoir having large cross sectional area when compared to cross sectional area of U – tube connected to it. For any change in pressure, change in the level of manometric liquid in the reservoir is small and change in level of manometric liquid in the U- tube is large.

To derive expression for pressure head at A:

BB and CC are the levels of manometric liquid in the reservoir and U-tube before connecting the point A to the manometer, writing gauge equation for the system we have,

+ y x S – h1 x Sm = 0

Sy = Smh1

Let the point A be connected to the manometer. B1B1 and C1 C1 are the levels of manometric liquid. Volume of liquid between BBB1B1 = Volume of liquid between

Let the point A be connected to the manometer. B1B1 and C1 C1 are the levels of manometric liquid. Volume of liquid between BBB1B1 = Volume of liquid between

CCC1C1

$$A\Delta = a\,h_2$$

$$\Delta = \frac{ah_2}{A}$$

Let 'h_A' be the pressure head at A in m of water.

$$h_A + (y + \Delta)\,S - (\Delta + h_1 + h_2)\,Sm = 0$$

$$h_A = (\Delta + h_1 + h_2)\,Sm - (y + \Delta)\,S$$

$$= \Delta\,Sm + \underline{h_1\,Sm} + h_2\,Sm - \underline{yS} - \Delta S$$

$$h_A = \Delta\,(Sm - S) + h_2\,Sm$$

$$h_A = \frac{ah_2}{A}\,(Sm - S) + h_2\,Sm$$

∴It is enough if we take one reading to get 'h_2' If '$\frac{a}{A}$' is made very small (by increasing 'A') then the I term on the RHS will be negligible.

$$\text{Then } h_A = h_2\,Sm$$

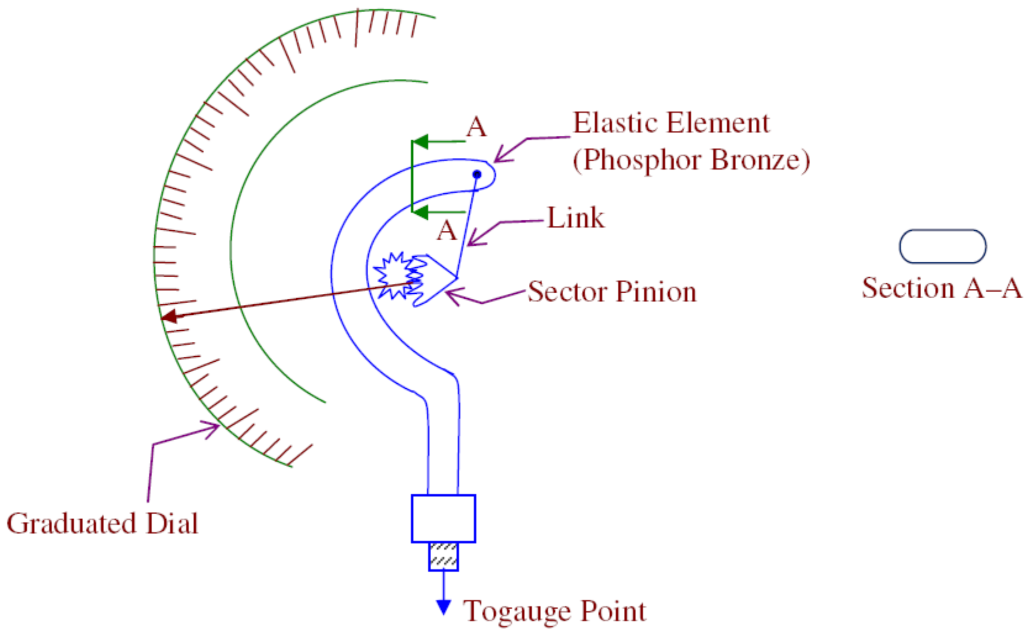
MECHANICAL GAUGES:

Pressure gauges are the devices used to measure pressure at a point.

They are used to measure high intensity pressures where accuracy requirement is less.

Pressure gauges are separate for positive pressure measurement and negative pressure measurement. Negative pressure gauges are called Vacuum gauges.

BASIC PRINCIPLE:



Mechanical gauge consists of an elastic element which deflects under the action of applied pressure and this deflection will move a pointer on a graduated dial leading to the measurement of pressure. Most popular pressure gauge used is Bordon pressure gauge.

The arrangement consists of a pressure responsive element made up of phosphor bronze or special steel having elliptical cross section. The element is curved into a circular arc, one end of the tube is closed and free to move and the other end is connected to gauge point. The changes in pressure cause change in section leading to the movement. The movement is transferred to a needle using sector pinion mechanism. The needle moves over a graduated dial.

... n to the subject :- FMHM

... matter can exist in solid state or fluid state. Fluid state can ... divided into liquid and gaseous states. In gener... same matter may exist in any of three stat... solid, liquid and gaseous states. Substances consist ... number of molecules separated by empty spac...

In solids, molecules are closely spaced, in a given volume solid is having large number of molecules. So in solids, the force of attraction between the molecules is large. Due to this high attraction forces movement of molecules is very less, so solid possess a rigid form.

In liquids, molecules are loosely packed, in a given volume liquid is having less number of molecules. In liquids, due to loosely packed molecules the molecules can move freely in result the force of attraction between the molecules is less but they force is sufficient to keep the liquid together in a definite volume.

In gases, molecules are very loosely packed, in a given volume gases are having very less number of molecules. So, gases have greater freedom of movement in result force of attraction between molecules is much more less

Fluid :- Fluid is a substance which is capable of flowing. It has no definite shape but it conforms to shape of container.

Fluid Mechanics :- Fluid mechanics is that branch of science which deals with the behaviour of the fluids (both liquids and gases) at rest as well as in motion.

This is deals with statics, kinematics and dynamics aspects of fluids.

- The study of fluids at rest is fluid statics.
- The study of fluids in motion, where pressure forces are not considered is fluid kinematics.
- The study of fluids in motion, where pressure forces are considered is fluid dynamics.

Properties of Fluids :-

Density or Mass Density :- Mass per unit volume is called density. The density of liquids may be considered as constant while that of gases changes with the variation of pressure and temperature.

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}$$

density for water is $1\ gm/cm^3$

or

$1000\ kg/m^3$.

Specific Weight or Weight Density:- It is defined as ratio between the weight of a fluid to its volume.

$$w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}}$$

$$w = \frac{\text{Mass of fluid} \times g}{\text{volume of fluid}} \qquad \left[\because \frac{\text{Mass}}{\text{volume}} = \rho\right]$$

$$\boxed{w = \rho g}$$

Specific weight for water is 9.81×1000 N/m^3 in SI

Specific Volume :- It is defined as volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume.

$$\text{specific volume} = \frac{\text{volume of fluid}}{\text{Mass of fluid}} = \frac{1}{\frac{\text{Mass}}{\text{volume}}} = \frac{1}{\rho}$$

Specific gravity(s):- It is defined as ratio of weight density of a fluid to weight density of a standard fluid. It is also called as relative density. For liquids, standard fluid is water, for gases, standard fluid is air.

$$s\,(\text{for liquids}) = \frac{\text{weight density of liquid}}{\text{weight density of water}}$$

$$s\,(\text{for gases}) = \frac{\text{weight density of gas}}{\text{weight density of air}}$$

weight density of liquid $= s \times$ wt density of water

$= s \times 1000 \times 9.81$ N/m^3

density of liquid = S× Density of water
= S× 1000 kg/m³.

Problem :- One litre of crude oil weighs 9.6 N. Calculate its specific weight, density and specific gravity.

Sol :- volume = 1 litre = $\frac{1}{1000}$ m³.

weight = 9.6 N.

Specific weight, $w = \frac{\text{Weight}}{\text{volume}} = \frac{9.6}{\left(\frac{1}{1000}\right)} = 9600\ N/m^3$.

density, $\rho = \frac{w}{g} = \frac{9600}{9.81} = 978.59\ kg/m^3$.

Specific gravity, $S = \frac{\text{Density of liquid}}{\text{Density of water}}$

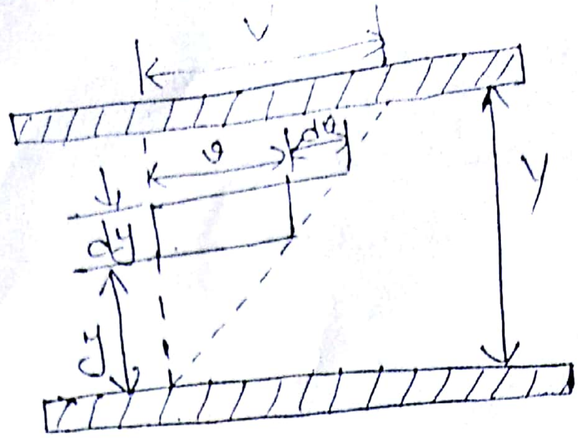
$S = \frac{978.59}{1000}$

$S = 0.978$ //.

Viscosity :- Viscosity is the property of fluid by virtue of which it offers resistance to the movement of one layer of fluid over an adjacent layer.

Viscosity is primarily due to cohesion and molecular momentum exchange between fluid layers and as flow occurs, these effects appear as shearing stresses between the moving layers of fluid.

Consider two plates at a distance y and the space between them filled with fluid. The lower plate is at stationary and upper plate is moving with a velocity V, by application of force F, area of plate is A.



the velocity v at a distance y from the lower plate will vary uniformly from zero to V(i.e from lower plate to upper plate).

~~From Experiments~~

While fluid is flowing top layer causes shear stress on the adjacent lower layer ~~causes~~ while lower layer causes shear stress on adjacent top layer. This shear stress is proportional to rate of change of velocity with respect to y.

$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}$$

μ. is coefficient of dynamic viscosity.

$\frac{du}{dy}$ is shear strain/rate of shear deformation/ velocity gradient.

$$\mu = \frac{\tau}{\left(\frac{du}{dy}\right)}$$

Viscosity can also be defined as shear stress required to produce unit rate of shear strain.

Units for dynamic viscosity:- (4)

MKS unit : $\frac{kgf\text{-}sec}{m^2}$

CGS unit : $\frac{dyne\text{-}sec}{cm^2}$ = poise

SI unit : $\frac{Ns}{m^2}$

$1 \frac{Ns}{m^2} = 10$ poise.

Units for kinematic viscosity:-

Kinematic viscosity is ratio between dynamic viscosity and density of fluid.

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho}.$$

MKS & SI unit : $\frac{m^2}{s}$.

CGS unit : cm^2/s = 1 stoke $= 10^{-4}\ m^2/s$.

Newton's law of viscosity:-

This law states that shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain.

$$\tau = \mu \frac{du}{dy}$$

Fluids which obey this relation are categorized as Newtonian fluids and fluids which do not obey are categorized as Non-Newtonian fluids.

Variation of viscosity with Temperature:-

The viscosity of liquids decreases with increasing temperature while the viscosity of gases increasing with the increase of temperature. Reason is that the viscous forces in a fluid are due to cohesive force and molecular momentum transfer.

~~In liquids, due to closely packed molecules~~

In liquids, cohesive forces are more. ~~con~~ Due to closely packed molecules cohesive forces will be reduced with the increasing temperature in result viscosity will be decreased.

In gases, molecular momentum transfer is high. With the increase in temperature, molecular momentum transfer increases and hence viscosity increases.

Relation b/w viscosity and temperature:

$\mu \uparrow$ with $T \uparrow$ for gases

$\mu \downarrow$ with $T \uparrow$ for liquids

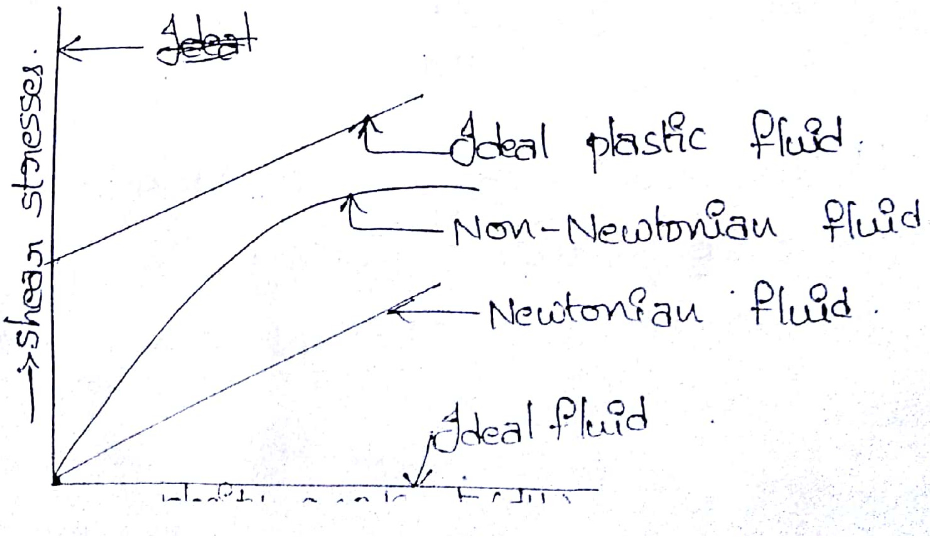
(i) For liquids,

$$\mu = \mu_0 \left[\frac{1}{1 + \alpha t + \beta t^2} \right]$$

(ii) For gas,

$$\mu = \mu_0 + \alpha t - \beta t^2.$$

Types of Fluids:- Fluids are classified into 5 types.



Ideal Fluid :- A fluid, which is incompressible and is having no viscosity. Ideal fluid is only an imaginary fluid.

Real Fluid :- A fluid, which possesses viscosity. All fluids are real fluids.

Newtonian Fluid :- A real fluid, in which shear stress is directly proportional to velocity gradient.

Non-Newtonian Fluid :- A real fluid, in which shear stress is not proportional to velocity gradient.

Ideal Plastic Fluid :- A fluid, in which shear stress is more than yield value and shear stress is proportional to velocity gradient.

Problem :-

The velocity distribution for flow over a flat plate is given by $u = \frac{3}{2}y - y^{3/2}$, where u is the point velocity in m/s at a distance y m. above the plate. Determine the shear stress at y = 9 cm. Assume dynamic viscosity as 8 poise.

Sol :-

$$u = \frac{3}{2}y - y^{3/2}$$

$$\frac{du}{dy} = \frac{3}{2} - \frac{3}{2}y^{1/2}$$

$y = 9\text{ cm} = 0.09\text{ m}$

$$\left.\frac{du}{dy}\right|_{y=0.09m} = \frac{3}{2} - \frac{3}{2}(0.09)^{1/2}$$

$$\left.\frac{du}{dy}\right|_{y=0.09m} = 1.05$$

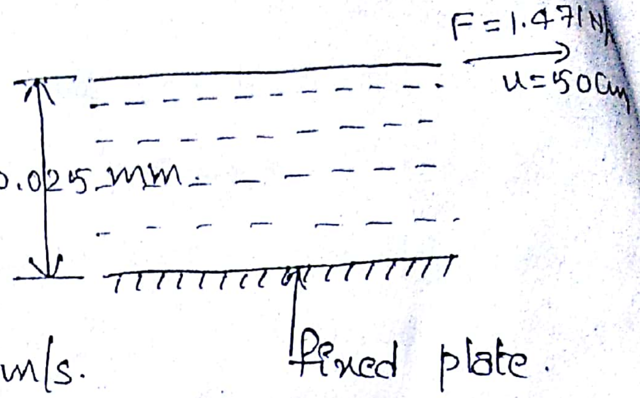
From Newton's law of viscosity, $\tau = \mu \frac{du}{dy}$.

$\mu = 8 \text{ poise} = 0.8 \text{ Ns/m}^2$.

$\tau = 0.8 \times 1.05 = 0.84 \text{ N/m}^2$ //.

Problem :- A plate 0.025 mm distant from a fixed plate moves at 50 cm/s and requires a force of 1.471 N/m² to maintain this speed. Determine the fluid viscosity b/w plates in poise.

Sol :-



Distance b/w plates,

$dy = 0.025$ mm.

$dy = 0.025 \times 10^{-3}$ m

velocity of upper plate, $u = 50$ cm/s.

Force on upper plate, $F = 1.471$ N/m²

From Newton's law of viscosity, $\tau = \mu \frac{du}{dy}$.

$du = u - 0 = 50$ cm/s $= 0.5$ m/s.

$dy = 0.025 \times 10^{-3}$ m

$\tau = 1.471$ N/m²

$\tau = \mu \frac{du}{dy}$

$1.471 = \mu\left(\frac{0.5}{0.025 \times 10^{-3}}\right)$

$\mu = \frac{1.471 \times 0.025 \times 10^{-3}}{0.5}$

$\mu = 0.07355 \times 10^{-3}$ Ns/m².

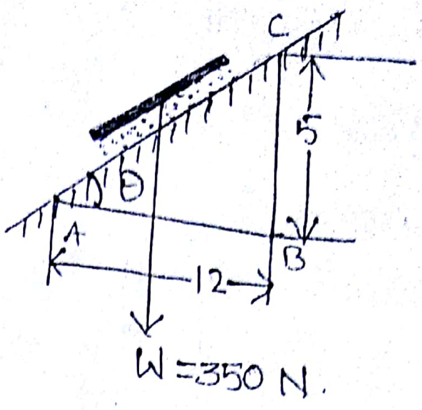
$\mu = 7.355 \times 10^{-5}$ Ns/m²

$\mu = 7.355 \times 10^{-4}$ poise.//

Problem :- A square plate of size 1m × 1m and weighing 350 N slides down an inclined plane with a uniform velocity of 1.5 m/s. The inclined plane is laid on a slope of 5 vertical to 12 horizontal and has an oil film of 1 mm thickness. Calculate the dynamic viscosity of oil.

$\tan\theta = \frac{5}{12}$ (13)

$\theta = 22.61°$.



Area of plate, $A = 1 \times 1$

$A = 1\ m^2$

Velocity of plate, $u = 1.5$ m/s.

thickness, $dy = 1$ mm.

Component of weight along the plane, $F = W \sin\theta$

$F = 350 \times \sin 22.6$

$F = 134.559$ N

$F \approx 134.6$ N

$du = u - 0 = 1.5$ m/s.

$\tau = \mu \frac{du}{dy}$.

$\frac{F}{A} = \mu \frac{du}{dy}$

$\Rightarrow \mu = \frac{F \times dy}{A \times du}$

$\mu = \frac{134.6 \times 1 \times 10^{-3}}{1 \times 1.5}$

$\mu = 89.733 \times 10^{-3}$ Ns/m²

$\mu = 89.733 \times 10^{-2}$ poise

$\mu = 0.897$ poise //.

Problem :-

Two large plane surfaces are 2.4 cm apart. The space between the surfaces is filled with glycerine. What force is required to drag a very thin plate of surface area 0.5 m² b/w two large plane surfaces at a speed of 0.6 m/s, if:

i) the thin plate is in the middle of two plane surfaces,

ii) the thin plate is at a distance of 0.8 cm from one

of the plane surfaces ?. Take dynamic viscosity of gly...
is 8.1×10^{-1} Ns/m^2.

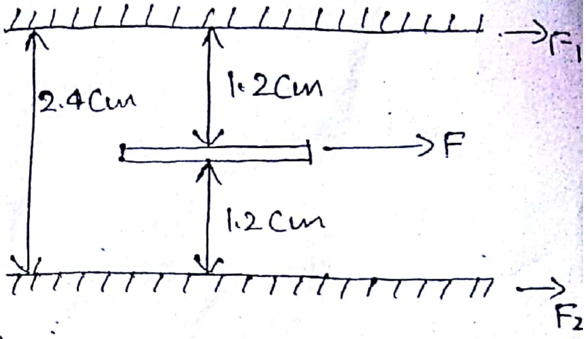
Sol:- Distance b/w two large surfaces = 2.4 cm.

Area of thin plate, $A = 0.5\ m^2$.

velocity of thin plate, $u = 0.6\ m/s$.

viscosity of glycerine, $\mu = 8.1\times10^{-1}\ Ns/m^2$.

Case (i):-



Total force required to drag the plate = shear force on upper side + shear force on lower side.

$$F = F_1 + F_2.$$

τ_1 be shear stress on upper side.

$$\tau_1 = \mu\left(\frac{du}{dy}\right)_1$$

$du = 0.6\ m/s$.

$dy = 1.2\ cm = 0.012\ m$.

$$\tau_1 = 8.1\times10^{-1}\left(\frac{0.6}{0.012}\right)$$

$$\tau_1 = 40.5\ N/m^2.$$

$$F_1 = \tau_1 \times A.$$

$$F_1 = 40.5\times0.5$$

$$F_1 = 20.25\ N.$$

τ_2 be shear stress on lower side.

$$\tau_2 = \mu\left(\frac{du}{dy}\right)_2 = 8.1\times10^{-1}\left(\frac{0.6}{0.012}\right)$$

$$\tau_2 = 40.5\ N/m^2$$

$$F_2 = \tau_2\times A = 40.5\times0.5$$

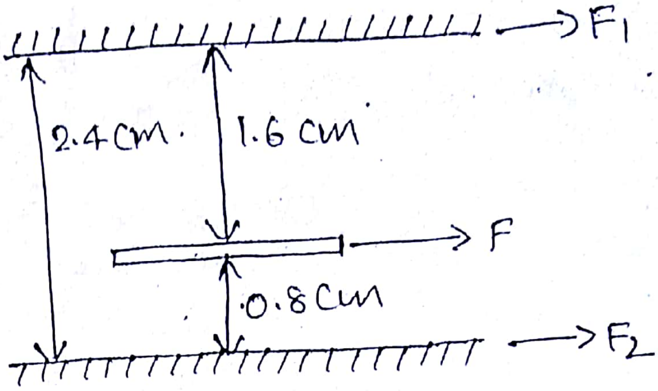
$$F_2 = 20.25\ N$$

$\therefore$ Total force, $F = F_1 + F_2$

$F = 20.25 + 20.25$

$\boxed{F = 40.5\ N}$

(ii):



...nce of plate from upper surface, $dy_1 = 0.016\ m$.

$\tau_1 = \mu\left(\frac{du}{dy}\right)_1$

$\tau_1 = 8.1\times10^{-1}\left(\frac{0.6}{0.016}\right)$

$\tau_1 = 30.375\ N/m^2$

$F_1 = \tau_1 \times A = 30.375 \times 0.5$

$F_1 = 15.18\ N.$

distance of plate from lower surface, $dy_2 = 0.8\times10^{-2}\ m.$

$\tau_2 = \mu\left(\frac{du}{dy}\right)_2$

$\tau_2 = 8.1\times10^{-1}\left(\frac{0.6}{0.8\times10^{-2}}\right)$

$\tau_2 = 60.75\ N/m^2$

$F_2 = \tau_2 \times A = 60.75 \times 0.5$

$F_2 = 30.375\ N.$

$\therefore$ Total force, $F = F_1 + F_2$

$F = 15.18 + 30.375$

$\boxed{F = 45.555\ N}$ //

Compressibility and Bulk Modulus:-

Bulk Modulus:- (K):- Bulk modulus is a measure incremental change in pressure dp which takes when a volume V of the fluid is changed by an incremental amount dv.

$$K = \frac{\text{change in pressure}}{\left(\frac{\text{change in volume}}{\text{original volume}}\right)} = -\frac{dp}{\left(\frac{dv}{V}\right)}$$

Compressibility:- Compressibility is reciprocal of the bulk modulus.

$$\text{Compressibility} = \frac{1}{K}.$$

Surface Tension:-

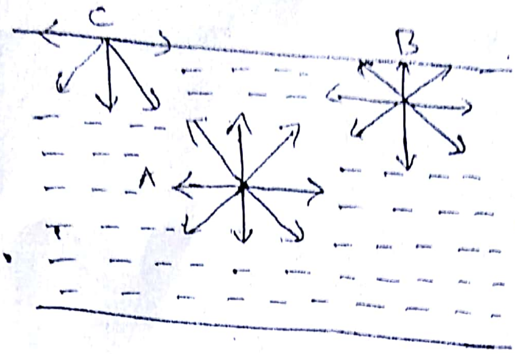
A liquid molecule on the interior of the liquid body has other molecules on all sides of it, so that the forces of attraction are in equilibrium.

A liquid molecule at the surface of the liquid does not have any liquid molecule above it, consequently there is a net downward force on the molecule due to the attraction of the molecules below it. This force on the molecules at the liquid surface, is normal to the liquid surface.

Due to this attraction of liquid molecules below the surface, a film or special layer seems to form on the liquid at the surface, which is in tension and small loads can be supported over it.

A small needle placed on water surface will not ... but will be supported by the tension at water ...ce.

The property of the liquid surface film to exert ...sion is Surface tension.



Surface Tension on Liquid Droplet :-

Consider a small spherical droplet of a liquid of ...adius 'r'.

Let

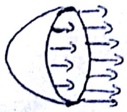
σ = Surface tension of liquid

p = pressure intensity inside the droplet.

d = Diameter of droplet

Let the droplet is cut into two halves. the forces acting are:

i) tensile force due to surface tension acting around the circumference of the cut portion



$= \sigma \times$ circumference

$= \sigma \times \pi d$.

ii) Pressure force on the area $= p \times \frac{\pi d^2}{4}$.

under equilibrium condition,

$$p \times \frac{\pi d^2}{4} = \sigma \times \pi d$$

$$\boxed{p = \frac{4\sigma}{d}}$$

Surface Tension on a Hollow bubble:-

Hollow bubble in air has two surfaces in contact with air, one inside and other outside. Two surfaces are subjected to surface tension.

$$p \times \frac{\pi d^2}{4} = 2 \times \sigma \times \pi d$$

$$\boxed{p = \frac{8\sigma}{d}}$$

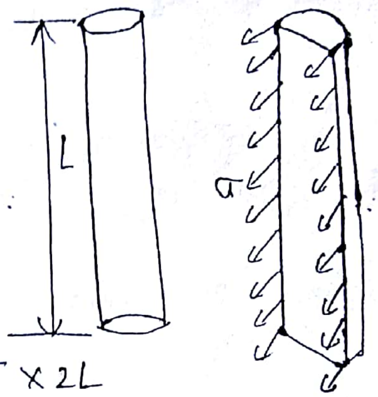
Surface Tension on a Liquid Jet :-

p = pressure intensity

σ = surface tension

pressure force = p × area

= p × L × d.



force due to surface tension = σ × 2L

$$p \times L \times d = \sigma \times 2L$$

$$p = \frac{\sigma \times 2L}{L \times d}$$

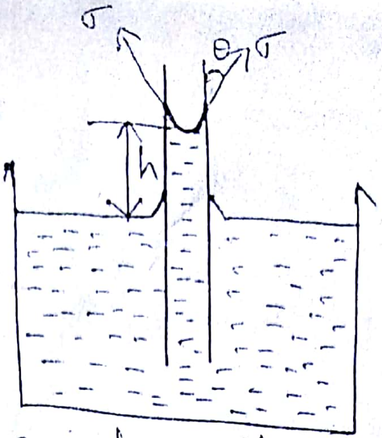
$$\boxed{p = \frac{2\sigma}{d}}$$

Capillarity :- It is a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid.

Rise of liquid surface is capillary rise.

Fall of liquid surface is capillary depression.

...ary Rise :-



...onsider a glass tube ... small diameter 'd' opened both ends and is ...serted in a liquid.

...e liquid will rise in the tube above the level of liquid.

...t h = height of liquid in tube.

Under equilibrium condition, weight of liquid of ...eight h is balanced by the force at the surface of ...iquid in tube.

But the force at the surface of liquid is due to surface tension.

Let

σ = Surface tension

θ = Angle of contact b/w liquid and glass tube.

The weight of liquid of height h in the tube =

$$(\text{Area of tube} \times h) \times \rho \times g$$

$$= \frac{\pi d^2}{4} \times h \times \rho \times g$$

Vertical component of the surface tensile force

$$= \sigma \times \pi d \times \cos\theta.$$

For Equilibrium,

$$\sigma \times \pi d \times \cos\theta = \frac{\pi d^2}{4} \times h \times \rho \times g$$

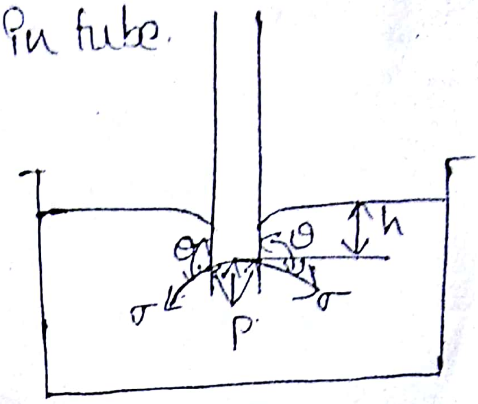
$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

θ is approximately equal to zero. $\Rightarrow \cos\theta = 1$

$$\Rightarrow \boxed{h = \frac{4\sigma}{\rho g d}}$$

capillary fall :- If the glass tube is dipped in mercury the level of mercury in the tube will be lower than the general level of outside liquid.

Let h = Height of depression in tube.



In downward direction,

force due to surface tension

$= \sigma \times \pi d \times \cos\theta$.

In upward direction

force due hydrostatic effect

$= p \times \frac{\pi d^2}{4}$

$= \rho g h \times \frac{\pi d^2}{4}$

For equilibrium,

$$\sigma \times \pi d \times \cos\theta = \rho g h \times \frac{\pi d^2}{4}$$

$$h = \frac{4\sigma\cos\theta}{\rho g d}.$$

for mercury and glass tube, $\theta = 128^\circ$.

Problem :-

Calculate the capillary rise in a glass tube of 3 mm diameter when immersed vertically in (a) water (b) mercury. Take surface tensions for mercury and water as 0.0725 N/m and 0.52 N/m respectively in contact with air. Specific gravity for mercury is 13.6.

Sol :- diameter of tube, $d = 3$ mm $= 3 \times 10^{-3}$ m.

σ for water $= 0.52$ N/m

σ for mercury $= 0.0725$ N/m.

∴ Density, $\rho = 13.6 \times 1000 = 13600\ kg/m^3$ (10)

Capillary rise for water ($\theta = 0°$).

$$h = \frac{4\sigma}{\rho g d} = \frac{4 \times 0.52}{1000 \times 9.81 \times 3 \times 10^{-3}}$$

$$h = 0.07\ m.$$

for mercury, $\theta = 128°$

$$h = \frac{4\sigma \cos\theta}{\rho g d} = \frac{4 \times 0.0725 \times \cos(128°)}{13600 \times 9.81 \times 3 \times 10^{-3}}$$

$$h = -4.46 \times 10^{-4}\ m$$

Vapour Pressure:-

A change from liquid state to gaseous state is vaporization.

Consider a liquid which is confined in a closed vessel. Increase the temperature of liquid upto vaporisation has occurs. When vaporization takes place the molecules escapes from the free surface of the liquid. These vapour molecules get accumulated in between free liquid surface and top of vessel. The pressure exerted by vapour molecules on liquid surface. This pressure is vapour pressure.

Cavitation:-

Consider a flowing liquid in a system. If the pressure at any point in this flowing liquid becomes less than or equal to vapour pressure.. the vaporization of the liquid starts. These vapour bubbles are carried by flowing liquid into high pressure region where they collapse, giving rise into high impact pressure. The pressure

The pressure developed by the collapsing bubbles so high that the material of the boundaries get eroded and cavities are formed on them. This phenomenon is known as cavitation.

Pressure and its Measurement

Pressure at a point :-

Consider fluid is at stationary. In that large mass of fluid consider a small area dA. dF is the amount of force exerted by surrounding fluid on dA in normal direction. Then the ratio of $\frac{dF}{dA}$ is intensity of pressure. Hence pressure at a point in a fluid at rest is

$$p = \frac{dF}{dA}$$

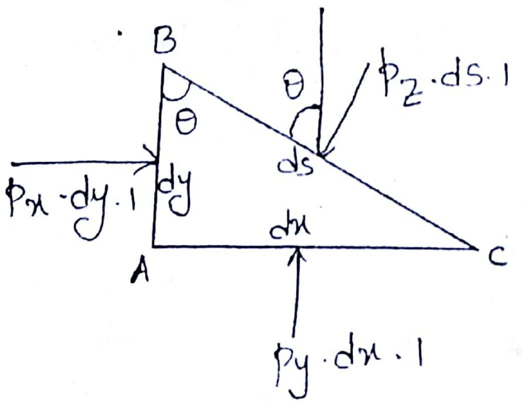
If the force F is uniformly distributed over the area A, then the pressure at any point is $p = \frac{F}{A}$.

$\therefore$ Force due to pressure $= p \times A$.

Pascal's law :-

It states that pressure or intensity of pressure at a point in a static fluid is equal in all direction

Consider one fluid element of dimensions dx, dy, dz



Let the width of the element is unity.

p_x, p_y and p_z are pressures acting on face AB, AC and BC.

Let $\angle ABC = \theta$.

Then the forces acting on the element are:

1. Pressure forces normal to the surfaces
2. Weight of element in vertical direction

Forces on the faces are:

Force on face AB $= p_x \times$ area of AB $= p_x \times dy \times 1$

Force on face AC $= p_y \times dx \times 1$

Force on face BC $= p_z \times ds \times 1.$

Weight of element $= M \times g$

$= V \times \rho \times g$

$= \left(\frac{AB \times AC}{2} \times 1\right) \times \rho \times g$

Resolving forces in

x-direction:

$$p_x \times dy \times 1 - p_z \times ds \times 1 \cos\theta = 0$$

From triangle ABC,

$$\cos\theta = \frac{AB}{BC}$$

$$\cos\theta = \frac{dy}{ds} \Rightarrow dy = ds\cos\theta.$$

$$\Rightarrow p_x\, dy - p_z\, dy = 0$$

$$\Rightarrow \boxed{p_x = p_z}$$

Resolving forces in y-direction:

$$p_y \times dx \times 1 - p_z \times ds \times 1 \sin\theta - \frac{dx \times dy}{2} \times 1 \times \rho \times g = 0$$

$$p_y \times dx - p_z\, ds \times \sin\theta - \frac{dx\,dy}{2} \times \rho g = 0.$$

From triangle ABC, $dx = ds\sin\theta$.

$$p_y\, dx - p_z\, dx = 0 \qquad [\because \text{weight is small.}]$$

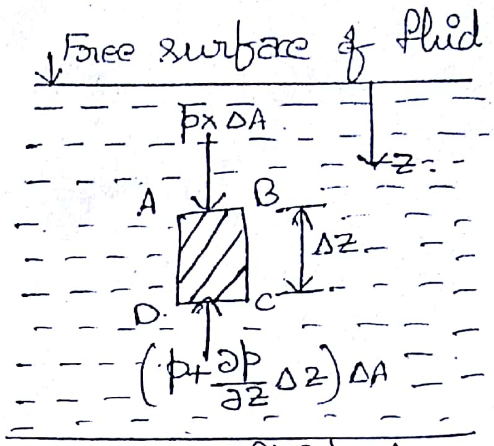
$$\Rightarrow \boxed{p_y = p_z}$$

$$\Rightarrow \boxed{P_x = P_y = P_z}$$

means pressure at any point is same in directions.

Hydrostatic law (Pressure Variation In a Fluid at rest):-

This law states that "rate of increase of pressure in a vertically downward direction must be equal to the specific weight of the fluid at that point.



Forces on a fluid element

Proof:-

Consider a small fluid element.

Let

ΔA = Cross-sectional area of element

ΔZ = Height of fluid element

p = Pressure on face AB

Z = Distance of fluid element from free surface.

Forces acting on the fluid element are:

1. Pressure force on AB = $p \times \Delta A$.
   This is acting perpendicular to face AB in downward direction.
2. Pressure force on CD = $\left[p + \frac{\partial p}{\partial z} \Delta z\right] \times \Delta A$
   This is acting perpendicular to face CD in upward direction.
3. Weight of fluid element = Density x Volume x g.
   $= \rho g (\Delta A \times \Delta Z)$.
4. Pressure forces on AD and BC are equal and opposite.

For equilibrium of fluid element

$$p \Delta A - \left(p + \frac{\partial p}{\partial z} \Delta z\right) \Delta A + \rho g (\Delta A \times \Delta Z) = 0.$$

$$\Rightarrow -\frac{\partial p}{\partial z} \Delta z \Delta A + \rho g \Delta A \Delta z = 0$$

$$\Rightarrow \boxed{\frac{\partial p}{\partial z} = \rho g = w}$$

This is Hydrostatic law.

$$\frac{dp}{dz} = \rho g$$

$$\int dp = \int \rho g \, dz$$

$$p = \rho g z$$

$$\Rightarrow z = \frac{p}{\rho g}$$

z is pressure head.

Absolute, Gauge, Atmospheric and Vacuum Pressures:-

Absolute Pressure:- It is the pressure which is measured with reference to absolute vacuum pressure.

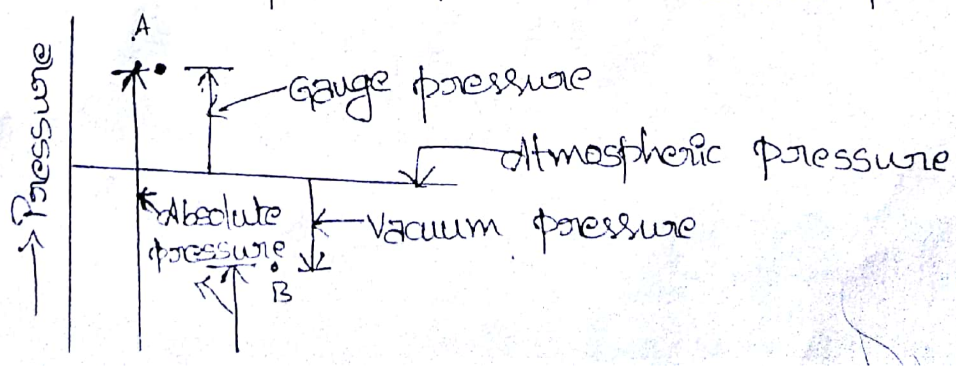
Gauge Pressure:- It is the pressure which is measured above the atmospheric pressure.

The atmospheric pressure on the scale is marked as zero.

Vacuum Pressure:- It is the pressure which is measured below the atmospheric pressure.

Absolute Pressure = Atmospheric pressure + Gauge Pressure.

Vacuum Pressure = Atmospheric pressure − Absolute pressure.



[Meas]urement of Pressure :-

Fluid pressure can be measured by
(1) Manometers
(2) Mechanical gauges.

Manometers :- Manometers are the devices used for measuring the pressure at a point in a fluid.
(a) Simple manometers
(b) Differential manometers.

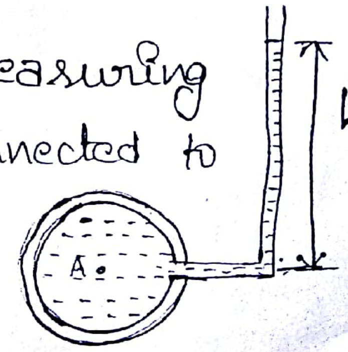
Simple Manometers :-

By using simple manometers, pressure at a point can be measured.

Simple manometers consisting a glass tube having one of its ends connected to a point where pressure is to be measured and other end is open to atmosphere.

Types of simple manometers :
(1) Piezometer
(2) U-tube manometer
(3) Single-Column manometer.

(1) **Piezometer :-** It is used for measuring gauge pressure. One end is connected to a point where pressure is to be measured and other end is open to atmosphere. The rise of liquid gives the pressure head at that point.



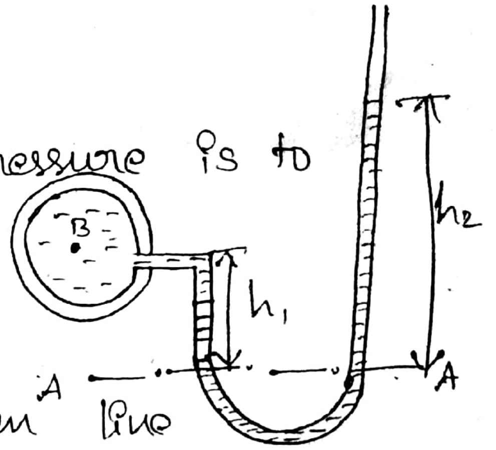
If at A, height of liquid is h.

Then pressure at A = $\rho \times g \times h \ \frac{N}{m^2}$.

2) U-tube Manometer :- It consists of a glass tube be in U-shape and it is having any other liquid whose specific gravity is more than specific gravity of liquid whose pressure is to be measured.

(a) For gauge pressure :-

B is the point at which pressure is to be measured.



Let

h_1 = height of light liquid above datum line

h_2 = height of heavy liquid above datum line

S_1 = specific gravity of light liquid.

S_2 = specific gravity of heavy liquid.

$\rho_1 = S_1 \times 1000$

$\rho_2 = 1000 \times S_2$.

Pressure is same for horizontal surface.

So pressure in left and right columns are same.

Pressure in left column = $P_B + \rho_1 g h_1$

Pressure in right column = $\rho_2 g h_2$

$$P_B + \rho_1 g h_1 = \rho_2 g h_2$$

$$P_B = \rho_2 g h_2 - \rho_1 g h_1.$$

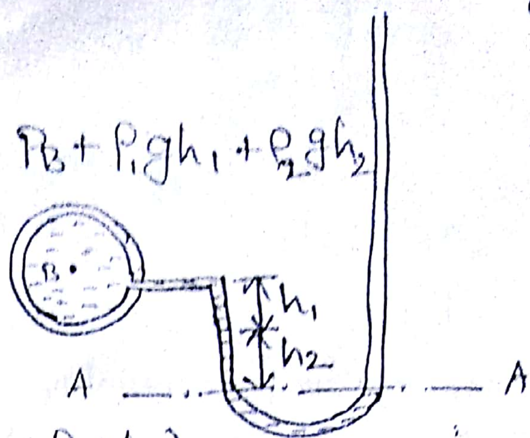
$$\boxed{P_B = g(\rho_2 h_2 - \rho_1 h_1)}$$

For vacuum pressure :-

(11)

Pressure above in left column = $P_B + \rho_1 g h_1 + \rho_2 g h_2$

Pressure in right column = 0



$$P_B + \rho_1 g h_1 + \rho_2 g h_2 = 0$$

$$P_B = -(\rho_2 g h_2 + \rho_1 g h_1).$$

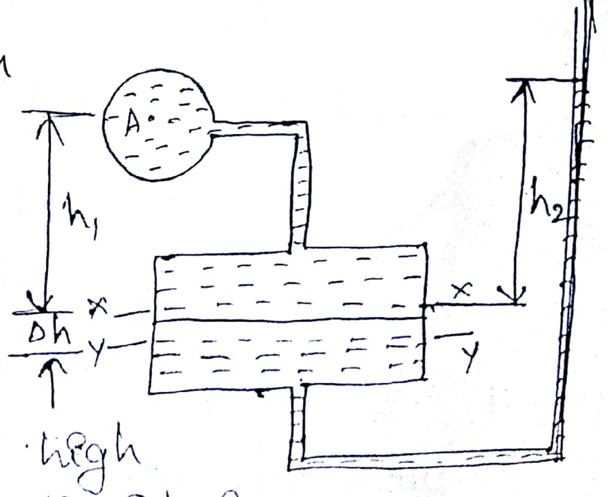
3) Single column Manometers :- These are modified form of U-tube manometers. These are having a reservoir with a large cross-sectional area than area of a tube connected to one of the limbs. Due to large cross-sectional area of reservoir for any variation in pressure liquid level change in reservoir is small which can be neglected, and pressure is given by height of liquid in other limb. The other limb may be vertical or inclined.

Types are :

(1) Vertical single column Manometer.
(2) Inclined single column Manometer.

1) Vertical single column Manometer :-

Let X-X be the datum line in reservoir and right limb, when it is not connected to the pipe.



When the manometer is connected to a pipe, due to high pressure P_A at A, the heavy liquid in reservoir will be pushed downward and will rise in right

Let

Δh = Fall of heavy liquid in reservoir

~~h_2 = height of~~

h_2 = Rise of heavy liquid in right limb

h_1 = Height of centre of pipe above X − X.

P_A = Pressure at A.

A = Area of reservoir

a = area of right limb.

S_1 = Sp. gr. of liquid in pipe

S_2 = Sp. gr. of heavy liquid in reservoir & right limb

ρ_1 = Density of liquid in pipe

ρ_2 = Density of liquid in reservoir.

Fall of heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb.

$$\therefore \quad A \times \Delta h = a \times h_2$$

$$\Delta h = \frac{a \times h_2}{A}$$

Now consider datum line Y − Y.

Pressure in right limb above Y − Y = $\rho_2 \times g \times (\Delta h + h_2)$

Equating pressures,

$$\rho_2 \times g \times (\Delta h + h_2) = \rho_1 \times g \times (\Delta h + h_1) + P_A$$

$$P_A = \rho_2 g (\Delta h + h_2) - \rho_1 g (\Delta h + h_1)$$

$$P_A = \Delta h [\rho_2 g - \rho_1 g] + \rho_2 g h_2 - \rho_1 g h_1$$

$$P_A = \frac{a \times h_2}{A} [\rho_2 g - \rho_1 g] + \rho_2 g h_2 - \rho_1 g h_1$$

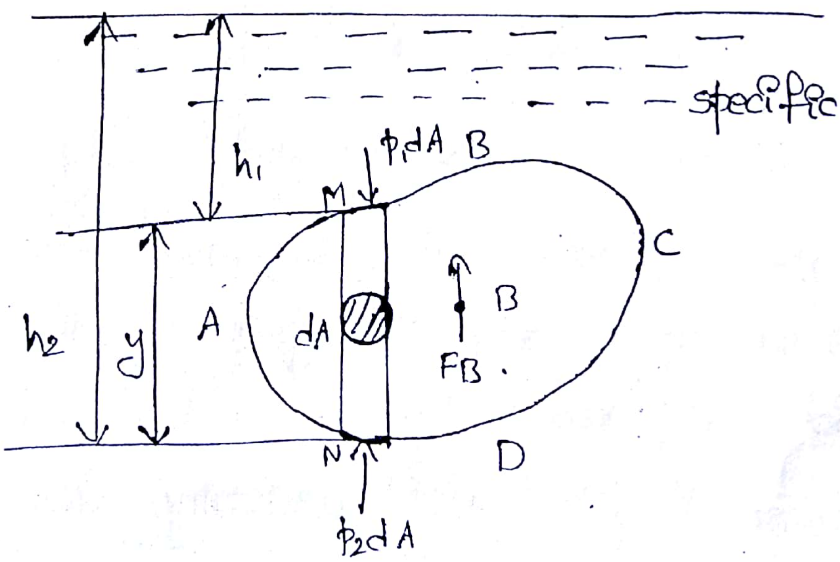
Buoyancy and Floatation. (1)

Buoyancy :-

When a body is immersed in a fluid either wholly or partially it is subjected to an upward force which tends to lift it up. This tendency for an immersed body to be lifted up in the fluid, due to an upward force which is opposite to the action of gravity, this upward force is known as "buoyancy".

- The force tending to lift up the body is known as buoyant force or force of buoyancy or upthrust.
- The point of application of the force of buoyancy on the body is known as "centre of buoyancy".
- Buoyant force can be determined by Archimedes principle which states that "when a body is immersed in fluid wholly or partially, it is buoyed or lifted up by a force which is equal to the weight of the fluid displaced by the body".

Proof:-



Consider a wholly submerged body ABCD. Horizontal pressure is equal and in opposite directions.

Consider an elementary vertical prism MN of c... section dA.

$P_1 dA$ is pressure acting on top of prism in down...
$P_2 dA$ is pressure acting on bottom in upward di...

$P_1 = wh_1$, $P_2 = wh_2$ $\quad \because h_2 - h_1 = y$.

w is specific weight & $P_2 > P_1$.

the difference between the upward and downward pressure forces is a net upward force which is equal to buoyant force dF_B on MN.

$$dF_B = (P_2 dA - P_1 dA)$$

$$= (wh_2 dA - wh_1 dA)$$

$$dF_B = w(h_2 - h_1) dA$$

$$dF_B = wy dA.$$

if $d\bar{V}$ is volume of prism MN, then $d\bar{V} = y dA$

then $dF_B = w d\bar{V}$.

Buoyant force F_B on entire submerged body ABCD is

$$F_B = \int dF_B = \int w d\bar{V} = w\bar{V}.$$

$\bar{V}$ is volume of submerged body

This Eqn indicates that buoyant force is exerted on a submerged body is equal to the weight of the fluid displaced by the submerged body.

The buoyant force acts vertically upwards through the centre of buoyancy which is coinciding with centroid of volume of fluid displaced.

For wholly submerged body centre of buoyancy

$= W \times BM \times \theta$ $\{\because F_B = W\}$

But these two couples are same. Hence Equate.

$W \times BM \times \theta = \int 2\rho g x^2 \theta \, L dx$

$W \times BM \times \theta = 2\rho g \theta \int x^2 L dx$

$W \times BM = 2\rho g \int x^2 L dx$

Ldx = Elemental area on water line = dA.

$W \times BM = 2\rho g \int x^2 dA$.

$2\int x^2 dA (= I)$ is 2nd moment of area of plan of the body at water surface about Y-Y axis.

$\therefore W \times BM = 2\rho g 2\int x^2 dA$

$W \times BM = 2\rho g I$

$BM = \frac{\rho g I}{W}$

W = weight of body

= weight of fluid displaced by body.

= ρg × volume of fluid displaced by body

= ρg × volume of body submerged in water

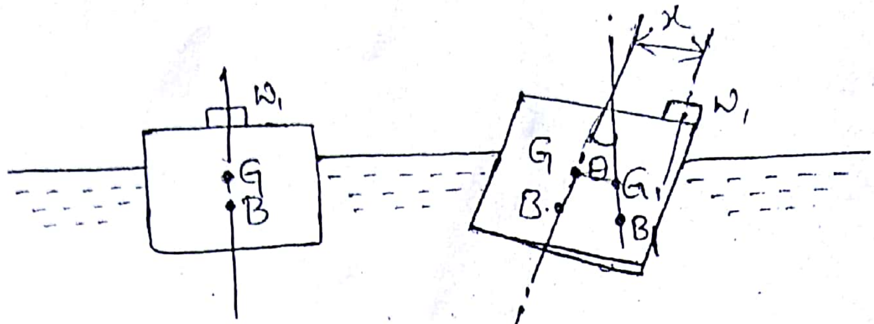
$W = \rho g \times \forall$

$BM = \frac{\rho g I}{\rho g \times \forall} = \frac{I}{\forall}$.

$GM = BM - BG = \frac{I}{\forall} - BG$.

$\therefore$ Meta-centric Height, $\boxed{GM = \frac{I}{\forall} - BG}$.

Experimental Method of determination of Meta-centric



Floating body / tilted body

Meta-centric height can be determined by providing centre of gravity.

Let,

W = weight of vessel including w_1

G = C.G of vessel

B = C.B of vessel

Weight w_1 is moved across the vessel towards right through a distance x.

Tilt the vessel through the angle θ.

As the weight has moved towards right, C.G and C.B also shifted to right.

the moment caused by the movement of load w_1 through a distance x must be equal to the moment caused by the shift of C.G from G to G_1.

Moment due to change of G $= GG_1 \times W$

$= W \times GM \tan\theta$

Moment due to movement of $w_1 = w_1 \times x$

$\therefore w_1 x = W GM \tan\theta$.

$$\Rightarrow \boxed{GM = \frac{w_1 x}{W \tan\theta}}$$

Stability:-

If any body is said to be stable if it comes back to its original position after a slight disturbance. Relative position of C.G and C.B of a body determines the stability.

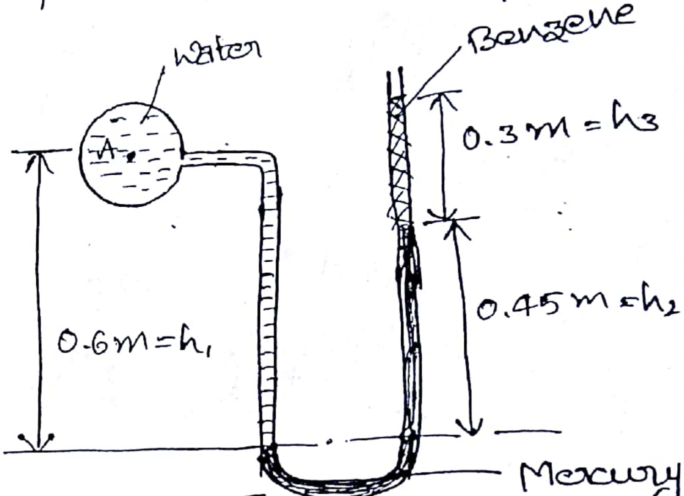
Stability of a Sub-merged Body:-

For a completely sub-merged body, position of centre of gravity and centre of buoyancy are fixed.

(22)

Problem :- The left leg of a U-tube mercury manometer is connected to a pipe-line conveying water, the level of mercury in the leg being 0.6 m below the center of pipe line, and the right leg is open to atmosphere. The level of mercury in the right leg is 0.45 m above the in the left leg and the space above mercury in the right leg contains Benzene (specific gravity 0.88) to a height of 0.3 m. Find the pressure in the pipe.

Solution :-



Pressures in left leg:

$$P_A + \rho_1 g h_1 = P_A + (1000 \times 9.81 \times 0.6)$$

$$= P_A + 5886$$

Pressures in right leg:

$$\rho_2 g h_2 + \rho_3 g h_3 = [(13.6 \times 0.45) + (0.88 \times 0.3)] \times 1000$$

$$= 6.384$$

Pressure head in left leg:

$$\frac{P_A}{w} + 0.6$$

Pressure head in right leg:

$$(13.6 \times 0.45) + (0.88 \times 0.3)$$

$$\Rightarrow \frac{P_A}{w} + 0.6 = (13.6 \times 0.45) + (0.88 \times 0.3)$$

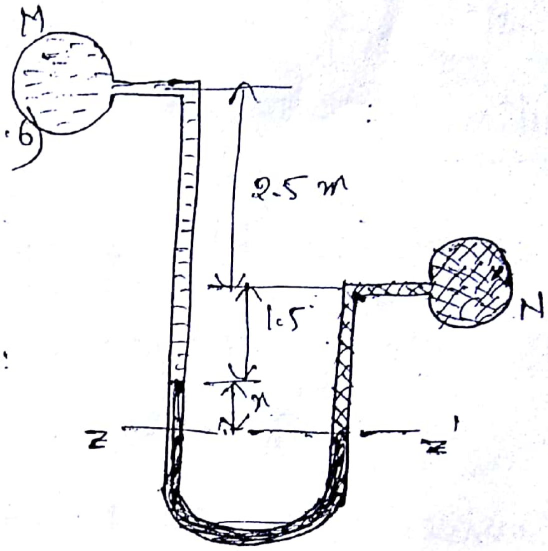
$$\frac{P_A}{w} = 5.784 \text{ m of water}$$

$$P_A = 5.784 \times 9.81 \times 1000$$

$$P_A = 5.674 \times 10^4 \text{ N/m}^2$$

Problem):- Pipe M contains carbon tetrachloride of specific gravity 1.594 under a pressure of 1.05 kg/cm² and pipe N contains oil of specific gravity 0.8. If the pressure in the pipe N is 1.75 kg/cm² and the manometric fluid is mercury. Find the difference x between the levels of mercury.

Pressure head in left column:



$$= \frac{1.05\times10^4}{1000} + [(2.5+1.5)\times1.594] + (x\times13.6)$$

$$= 16.876 + 13.6x$$

Pressure head in right column:

$$= \frac{1.75\times10^4}{1000} + [(1.5+x)\times0.8]$$

$$= 18.7 + 0.8x.$$

Equating

$$\Rightarrow 16.876 + 13.6x = 18.7 + 0.8x.$$

$$12.8x = 1.824$$

$$x = 0.1425\ m$$

$$\boxed{x = 14.25\ cm}$$

Problem :- A wooden block of rectangular section 1.25 m wide, 2 m deep, 4 m long floats horizontally in sea water. If the specific gravity of wood is 0.64 and water weighs 1025 kg(f)/m³, find the volume of liquid displaced and the position of the centre of buoyancy.

Solution :-

A/c to Archimedes principle, w.k.t

weight of liquid displaced by body = weight of body

Weight of block = volume × density

$= (1.25 \times 2 \times 4) \times (0.64 \times 1000)$

$= 6400$ kg

Volume of sea water displaced by the block $= \frac{6400}{1025}$

$= 6.24\ m^3$.

Let h be the depth of the block under water,

$4 \times 1.25 \times h = 6.24$

$h = \frac{6.24}{4 \times 1.25} = 1.248$ m.

∴ Centre of Buoyancy $= \frac{1.248}{2} = 0.624$ m. above base //

Problem :- A wooden cylinder of diameter d and length 2d floats in water with its axis vertical. Locate the metacentre with reference to water surface. Specific gravity of wood is 0.6.

Solution :-

weight of cylinder $= \frac{\pi d^2}{4} \times 2d \times 0.6 \times 9.81 \times 1000$

$= 9245.7 d^3$ N.

∴ Volume of water displaced by cylinder $= \frac{9245.40}{9810}$

$= 0.942 d^3$ $\left[\because V = \frac{W}{\rho g}\right]$

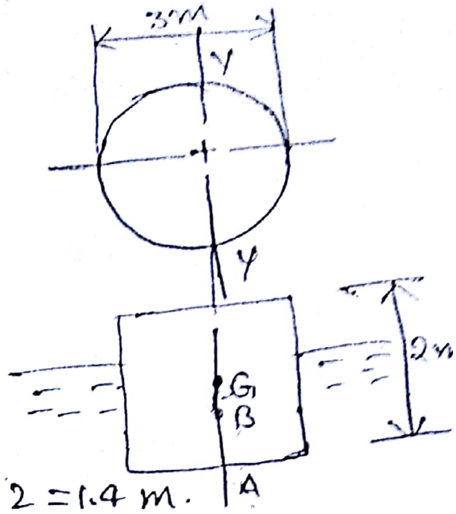
Depth of immersion $= \frac{0.942 d^3}{\frac{\pi d^2}{4}}$

$= 1.199 d \simeq 1.2 d$

∴ Height of C.B above the base $= \frac{1.2d}{2} = 0.6d$.

Problem:- A solid cylinder of diameter 3 m has a height of 2 m. Find the meta-centric height of the cylinder when it is floating in water with its axis vertical. The specific gravity of cylinder is 0.7.

Solution:-



Diameter of cylinder, $D = 3\,m$.

Height of cylinder, $h = 2\,m$

Specific gravity of cylinder $= 0.7$

Depth of immersion of cylinder $= 0.7 \times 2 = 1.4\,m$.

$$AB = \frac{1.4}{2} = 0.7\,m.$$

$$AG = \frac{2}{2} = 1\,m.$$

$$BG = AG - AB = 1 - 0.7 = 0.3\,m.$$

Meta-centric height, $GM = \frac{I}{V} - BG$.

I = Moment of Inertia about Y-Y axis.

$$I = \frac{\pi D^4}{64} = \frac{\pi}{64} \times (3)^4$$

V = Volume of cylinder in water

V = Area x depth of immersion.

$$V = \frac{\pi D^2}{4} \times 1.4$$

$$V = \frac{\pi}{4}(3)^2 \times 1.4.$$

$$GM = \frac{I}{V} - BG$$

$$GM = \frac{\frac{\pi D^4}{64}}{\frac{\pi D^2}{4} \times 1.4} - BG = \frac{D^2}{1.4(16)} - BG$$

$$GM = \frac{(3)^2}{(16) \times 1.4} - 0.3.$$

$GM = 0.1017\,m$

KINEMATIC FLUID FLOW

Fluid Mechanics is not a Subject it is a Science to study about the behaviours of the fluid particles at rest (or) in motion.

NOTE:-

Rest (Static)

Motion (Dynamic)

Static:-

The Fluid particle is at rest is called st-

Dynamic:-

The Fluid particle is in Motion is called Dy-

Dynamic is also classified in to two types.

(i) Kinetic (ii) Kinematic.

Kinetic:-

The Fluid particle is in motion with Consider the force is called Kinetic.

Kinematic:-

The Fluid particle is in Motion with out Consider the Force is called Kinematic.

Types of fluids:-

(i) Real Fluid - Water

(ii) Ideal Fluid - petrol, Diesel, etc.

(iii) New ton Fluid

(iv) Non - Newton Fluid

(v) Plastic Fluid - Paints.

Properties of Fluids:-

* weight density (Specific weight)
* Mass " (" mass)
* Volume (Specific Volume)
* Gravity
* Viscocity

* Weight density:- The weight of an object is the force of gravity on the object & may be defined as the mass time the acceleration of gravity.

$W = mg$. Since the weight is a force, it's S.I unit is the newton, Density is mass/volume.

Mass Density:- The density or more preciously the volumetric mass density of a substance is its mass per unit volume. The symbol must often u-sed for density is "ρ". although the Lat is letta "D" can also be used.

Volume:- Volume is the 3-D space enclosed by a closed surface volume is often quantified numer-ically using the S.I derived unit the cube meter

Gravity:- Gravity is force pulling together al matter. The more matter the more gravity so things that have a lot of matter such as planets moons, stars pull more strongly.

Viscosity :-

A quantity expressing the magnitude of internal friction in a fluid, as measured by the force per unit area resisting uniform flow.

Ex:- Honey has a highest viscosity by than water.

Fluid pressure :-

when a fluid is contained in a vessel it exerted force at all the points the force per unit area called pressure.

$$P = \frac{F}{A} = \text{pascal}.$$

Types of Fluid pressure :-

Absolute Pressure :-

is zero refered against a perfect vacum. so it is equal to gauge pressure + Atmosphere pressure. $P_{abs} = P_{atm} + P_G$

Vaccum pressure :-

Vaccum pressure is the difference between the atmospheric pressure and absolute pressure.

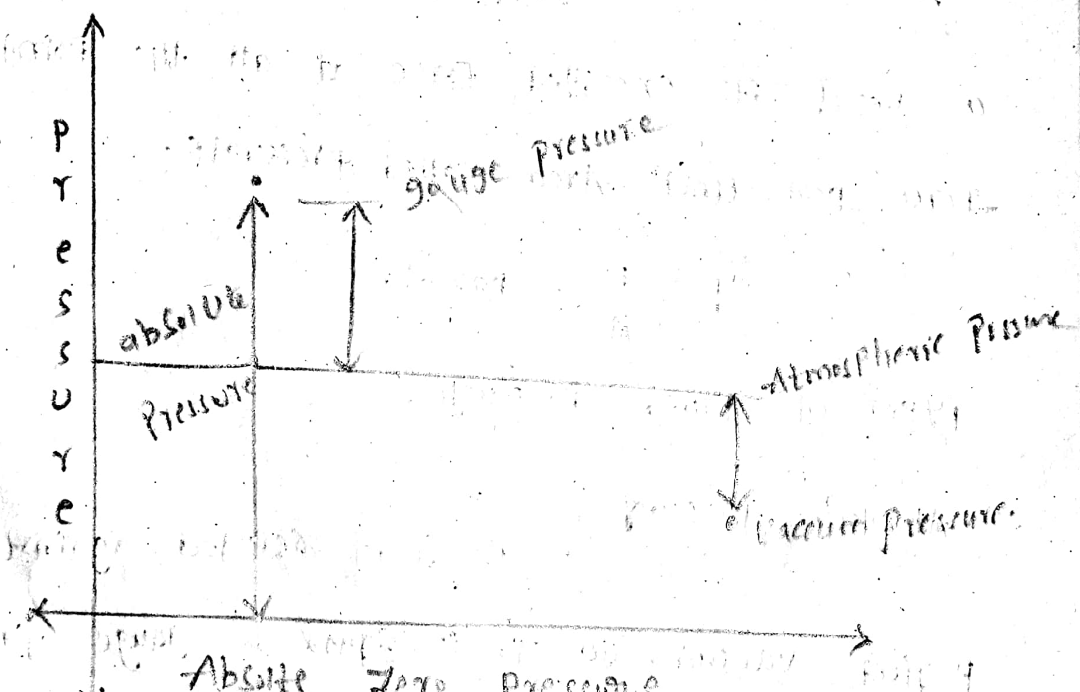
Gauge Pressure :-

The Amount by which the

pressure measured in a fluid exceed that of the Atmosphere. $P_G = P_{abs} - P_{atm}$.

Atmospheric Pressure:-

The pressure exerted by the earth's Atmosphere at any given point being the product of mass of the atmospheric column of the unit Area alone given point and of gravitational acceleration at given point.



PROBLEMS

1. A Rectangular tray Surface 3m deep & 2M wide it lies in vertical plane in water. Determine the total pressure and position of centre of pressure on the plane Surface when it is upper edge is horizontal.

Sol: Given data:-

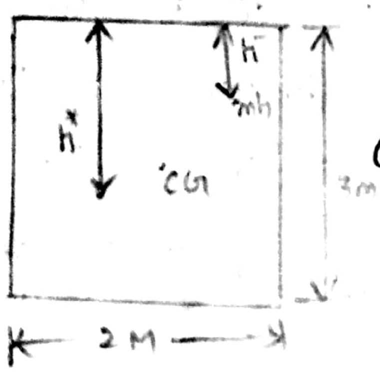
Height (H) = 3m

Base (b) = 2m

Total pressure $F = \rho g \bar{h} A$

$= 1000 \times 9.8 \times 1.5 \times (3 \times 2)$

$= 88290 N.$



$\bar{H} = \frac{1}{2}(3) = 1.5m$

Centre of Pressure $H^* = \frac{IG}{A \times \bar{h}} + \bar{h}$

$IG = \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5$

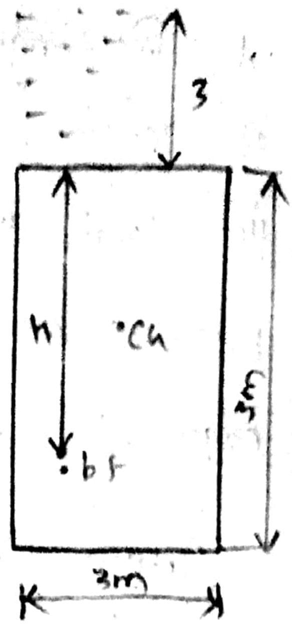
$H^* = \frac{4.5}{6 \times 1.5} + 1.5 = 2m$

2. A rectangular plane Surface is 3m deep & 2m wide it lies upper edge it's below the water surface at 3m. Determine the total pressure & position of centre of pressure on the plane surface when it's upper edge is horizontal.

G.D:-

Sol: deep = 3m

wide = 2m



Total pressure $F = \rho g \bar{h}$

$= 1000 \times 9.81 \times 4.5$

$= 44145\ N$

$\bar{H} = 3 + (\frac{3}{2}) = 4.5$

Centre of pressure $h^* = \frac{I_G}{A \times \bar{h}} + \bar{H}$

$I_G = \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5$

$H^* = \frac{4.5}{6 \times 4.5} + 4.5$

$H^* = 4.66\dot{6}$

3. When the plate of circular dia. 3m which is placed in water & vertical direction 3m that the centre of the plate 5m below the surface of water. Find the total pressure & position of centre of pressure.

Sol: diameter of circular plate (d) = 3m

$A = \frac{\pi}{4} d^2 = \frac{\pi (3)^2}{4} = \frac{9\pi}{4} = 7.0685$

Total pressure $F = \rho g h$

$= 1000 \times 9.8 \times 5.6366$

$= 55236.68\ N$

$\bar{H} = 5 + \frac{4(3)}{6\pi} = 6.2732$

$= 5 + 0.6366 = 5.6366$

Centre of Pressure $h^* = \frac{I_G}{A \times \bar{h}} + \bar{h}$

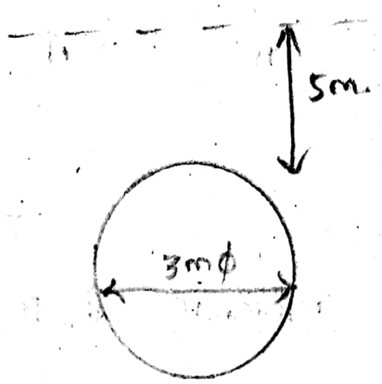
$= \frac{\frac{\pi(3)^4}{64}}{}$

$= 7.0685 \times 6.2732 + 6.2732$

$= 6.3628$

$= \frac{81\pi}{2549.96} + 5.6366$

$h^* = 5.6464.$



$A = 7.0685$

$\bar{H} = 3.5 + \frac{4d}{6\pi}$

$= 3.5 + \frac{4(3)}{6\pi}$

$= 3.5 + 1.2732 = 4.7732$

Total pressure $F = \rho g h$

$= 1000 \times 9.8 \times 4.7732 = 46777.36$

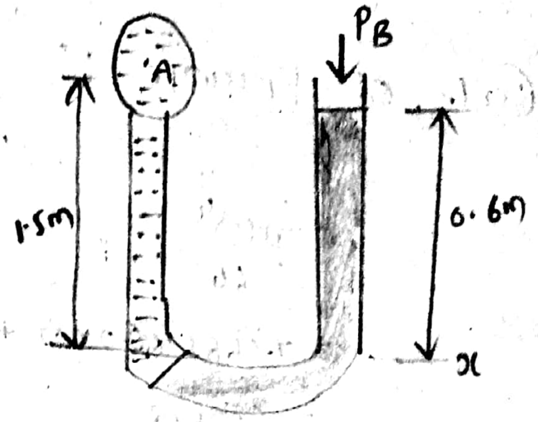
Centre of pressure $H^* = \frac{I_G}{A\bar{h}} + \bar{h}$

$I_G = \frac{\pi d^4}{64} = \frac{81\pi}{64} = 3.9760$

$H^* = \frac{3.9760}{7.0685 \times 4.7732} + 4.7732$

$= 4.8910$

1. Determine the pressure above the atmospheric for the manometer dimensions are shown in Fig.



Sol: Pressure of water P

pressure at taken height 1.5m

" " 'A' $= \rho_1 g_1 h_1 + P_A = 0$

$= 1000 \times 9.81 \times 1.5 + P_A$

$\Rightarrow P_A + 14715$

Pressure at 'B' $= \rho_2 g_2 h_2 + P_B$

$= 13,600 \times 9.81 \times 0.6 + 0$

$= 80049.6 + P_B$

$P_A + 14715 = P_B + 80.04$

$P_A - P_B = 80049.6 - 14715$

$P_A = 63345 \ N/mm^2$

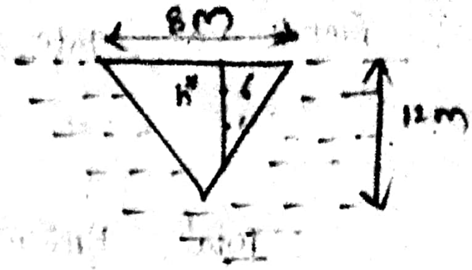
$= 63.34 \ kN/mm^2$

2. A Triangular plate of base 8m & altitude 12m when it's immersed vertically in water surface level of water. Find the total pressure and centre of pressure.

Sol: Area of Triangle $= \frac{1}{2} bh$

$= \frac{1}{2} \times 8 \times 12$

$= 48 m.$



Total pressure $F = \rho g A \bar{h}$

$\bar{h} = \frac{H}{3} = \frac{12}{3} = 4$

$= 1000 \times 9.81 \times 48 \times 4$

$= 1883520 N$

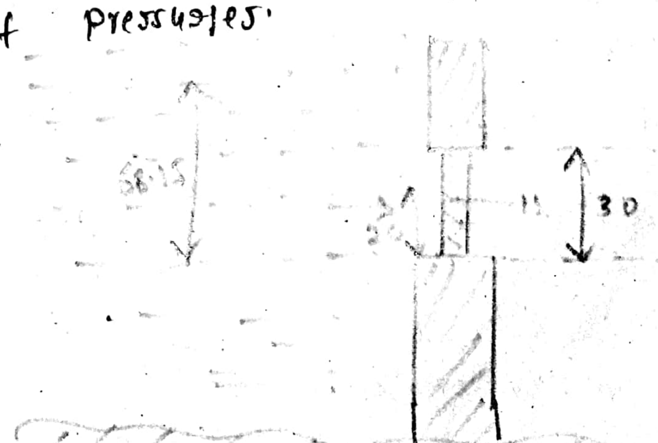
Centre of pressure $h^* = \frac{I_G}{A\bar{h}} + \bar{h}$

$I_G = \frac{bd^3}{36} = \frac{8 \times (12)^3}{36} = 384.$

$h^* = \frac{384}{48} + 4 = 6m.$

3. A vertical slice gate is used to cover an opening in a dam. The opening cover is 60 m wide and 30m Height up stream height 45m. & down stream touches the 45m & down stream touches the top gate. Find the total pressure & Centre of pressure & also Find Resultance of total and centre of pressures.

Sol:



Width of gate $b = 60m$

Depth of gate $d = 30m$

$\rho_1 = \rho_2 = 1000$

Total Pressure $F_1 = \rho_1 g_1 A_1 H_1$

$F_2 = \rho_2 g_2 A_1 H_1$

Area of gate $A_1 = A_2 = bd = 60 \times 30 = 1800 m^2$

$\bar{H_1} = 45 + \frac{30}{2} = 60m$

$\bar{H_2} = \frac{30}{2} = 15m$

$F_1 = 1000 \times 9.81 \times 1800 \times 60$

$= 1059480000N$

$F_2 = 1000 \times 9.81 \times 1800 \times 15$

$= 264870000N$

Centre of Pressure $h_1^* = \frac{I_G}{A \times \bar{H_1}} + \bar{H_1}$

$I_G = \frac{bd^3}{12} = \frac{60 \times (30)^3}{12} = 135000$

$= \frac{135000}{18000 \times 60} + 60 = 61.25$

$H_2^* = \frac{I_G}{A \times \bar{H_2}} + \bar{H_2}$

$= \frac{135000}{1800 \times 15} + 15$

$H_2^* = 20$